Where ignorance is bliss, 'tis folly to be wise - the value of information in contests*

Philipp G. Denter[†] and Dana Sisak[‡]

preliminary version

July 15, 2009

Abstract

We analyze a two-player Tullock contest with asymmetric valuations and one-sided asymmetric information. We give a general characterization of the possible equilibria arising in the Bayesian game. Using those results we show that a player may well prefer to stay ignorant over being informed about his opponent's type. In those situations she can use her ignorance as a strategic instrument to bind herself credibly in the contest to some effort that is not part of her best response, which puts her in a situation similar to that of a Stackelberg leader. We characterize conditions for this result to hold. Interestingly, if the contest is too discriminating staying ignorant is always weakly dominated. Thus, with respect to the incentives to acquire information we identify an important difference between Tullock contests and All-pay auctions.

Keywords: Contest, Asymmetric Information, Commitment, Value of Ignorance

JEL-Classification: D72, D82, L12

*This version is preliminary and any comments are very welcome. "Where ignorance is bliss, 'tis folly to be wise" is taken from from Thomas Gray's (1768) poem "Ode on a Distant Prospect of Eaton College". We are indebted to Stefan Bühler, Magnus Hoffmann and especially Martin Kolmar and Uwe Sunde for very helpful suggestions and comments. Of course, all errors remain our own.

[†]University of St.Gallen, Institute of Public Finance and Fiscal Law (IFF-HSG), Varnbuelstr. 19, CH-9000 St Gallen. eMail: philipp.denter@unisg.ch.

[‡]University of St.Gallen, Institute of Public Finance and Fiscal Law (IFF-HSG), Varnbuelstr. 19, CH-9000 St Gallen. eMail: dana.sisak@unisg.ch.

1 Introduction

Contests are frequently encountered in reality.[1] For example, political rent-seeking is inherently a contest.[2] In most situations we should expect that contestants are heterogenous in some sense and that there are some informational asymmetries between the players. This is in particular true if a new and yet unknown player enters the field.[3] In this paper we address this issue in detail and show how the information structure influences outcomes.

Specifically, we look at a Tullock contest where two players compete for some prize. Players are heterogeneous with respect to their valuation. The valuation of one player is common knowledge. The other player's valuation of the prize is her private knowledge and only the distribution is common knowledge. In this framework we first solve for all equilibria and identify conditions for corner solutions. Second, we show the interesting effect that the uninformed player might actually be better off without knowing the true type of her opponent. This is a new point in the contest literature. The intuition is that credibly staying ignorant enables the player to commit to some effort that could not be sustained in a full-information equilibrium, since this effort is not part of her best response. By committing to stay ignorant the player is able to commit to a particular action in a later stage. We show that in many instances the ignorant player can gain from this commitment opportunity, since she can choose her optimal effort in the contest so as to induce lower outlays of (some) of her opponents, thus effectively soften the competition as compared to the full-information game. Our finding is that if contestants are similar in expectation, it is more likely that ignorance is bliss. This finding contradicts common wisdom, which often states the strong players are those who do not care about their opponent(s), since they expect to win anyway. The reason that this is not true is the following: An uninformed player with intermediate valuation can discourage a potential weak opponent by choosing a high effort, compared to the full information equilibrium. This is credible because of the possibility to face a strong player instead. Similarly, this player can appease a potential strong opponent by choosing a comparatively low effort. Again, this is credible because she might face the weak opponent instead. We provide conditions under which this result holds. We show that if the contest is

[1] Garfinkel and Skaperdas (2007), Konrad (2009), and Sisak (2009) provide some very recent surveys over this literature.

[2] Congleton, Hillman, and Konrad (2008) gives an extensive overview over the rent-seeking literature.

[3] A recent example from politics is Barack Obama, who, though still being quite unknown to the public in the U.S., entered the democrats' competition for being a presidential candidate in the 2008 election. Interesting papers applying contests to the analysis of political competition are for example Konrad (2004) and Klumpp and Polborn (2006).

perfectly discriminating, in the parlance of auction theory an All-pay auction, staying ignorant is always weakly dominated.

The role of commitment in games is well known at least since von Stackelberg (1934). Usually, it is a strategic advantage if a player can move *first* because this allows her to commit to a particular action. The intuition for this advantage comes from the fact, that the player who moves first can *commit* to choose an action/strategy which is not in her own best-response correspondence, however, taking into account the best response of all other players. For example a Stackelberg leader in a Cournot oligopoly game chooses her output, taking into account the response of the followers, and so can gain a bigger profit and market share. In the the literature on contests Dixit (1987) found, that this result carries over to a two player Tullock contest. A player who has the opportunity to move first has an advantage, because she can commit to a particular level of effort. Depending on the relative abilities of the contestants, the leader's effort level possibly differs from the effort level in a full-information Nash equilibrium. In particular, if the leader is the favorite (underdog) of the game, she will spend more (less) than in Nash equilibrium.[4] The intuition is the following: in the Nash equilibrium the favorite's efforts are strategic substitutes to the underdogs efforts, whereas the underdogs efforts are strategic complements to the favorite's efforts. By committing to a higher (lower) effort, the favorite (underdog) can induce lower equilibrium efforts of their opponents. Interesting articles by Baik and Shogren (1992), Yildirim (2005), or Fu (2006) extent Dixit's analysis by introducing endogenous order of moves, multiple stages, or asymmetric information. In a more recent paper Morgan and Várdy (2007) analyze a sequential move Tullock contest. They come to the interesting and surprising result that if the follower has to pay some $\epsilon > 0$ in order to observe the leaders effort choice, the value of commitment is lost and in the unique equilibrium players choose their respective simultaneous-move Nash equilibrium efforts.

The papers most closely related to ours are Hurley and Shogren (1998b), Baik and Shogren (1995), Morath and Münster (2008), and Slantchev (2008). Baik and Shogren (1995) consider an asymmetric information contest and analyze how the opportunity to acquire informaion changes rent dissipation. Hurley and Shogren (1998b) discuss equilibria in two-player Tullock contests with one-sided asymmetric information and provide extensive comparative statics. Slantchev (2008) looks at a two-player contest with one-sided asymmetric information, where the type of the player with private information is drawn from a two-point distribution. In

[4] The terms 'favorite' and 'underdog' are due to Dixit (1987). A given player is called the favorite (underdog) if her equilibrium winning probability is larger (smaller) than 50 percent.

this context he discusses equilibria where all types or only one type of the unknown player are active. Morath and Münster (2008) analyze information acquisition in the context of All-pay auctions, where players do not know their own type and the type of their opponent. Each player can engage in acquiring the information about her own type. Depending on the cost of information acquisition only one player might invest in information. Therefore, one-sided asymmetric information can emerge *endogenously* in contests.

The paper is organized as follows. In the next section we shortly review standard results from full information Tullock contests as a benchmark. In section 3 we proceed to analyze the contest when there is one-sided asymmetric information with respect to the valuation of one player, player B. In section 4 we develop a set up to analyze the game when information acquisition is possible. In section 5 we solve the model for the case of perfectly observable information acquisition, in section 6 we show what changes if information acquisition is not perfectly observable. In section 7 we briefly discuss recent papers analyzing signalling in Tullock contests, which might serve as an alternative means for information transmission. Section 8 concludes.

2 Full Information: The benchmark case

Assume a standard two player Tullock contest where player $i = A, B$ spends effort x_i in order to win a prize to which he assigns a value v_i. Marginal costs are equal to one. We abstain from modeling the players heterogeneous with respect to their costs, to keep the analysis as simple as possible and to isolate the commitment effect in the next section. A general discussion with all important results in this benchmark case can be found in Konrad (2009).

Player i maximizes her expected utility in the full information (FI) game

$$\max_{x_i \in \mathbb{R}^+} EU_i^{FI} = v_i \, Pr_i[\text{win}] - x_i = v_i \, \frac{x_i}{x_A + x_B} - x_i \tag{1}$$

where $Pr_i[\text{win}] = 1 - Pr_j[\text{win}]$, $i = A, B$ and $i \neq j$. The individual best responses are single-valued and given by

$$x_i^*(x_j) = \max\{0, \sqrt{v_i x_j} - x_j\}. \tag{2}$$

Given the best response it is easily shown that the Nash equilibrium (NE) efforts are given by

$$x_i^{NE} = \frac{v_i^2 v_j}{(v_A + v_B)^2} > 0. \tag{3}$$

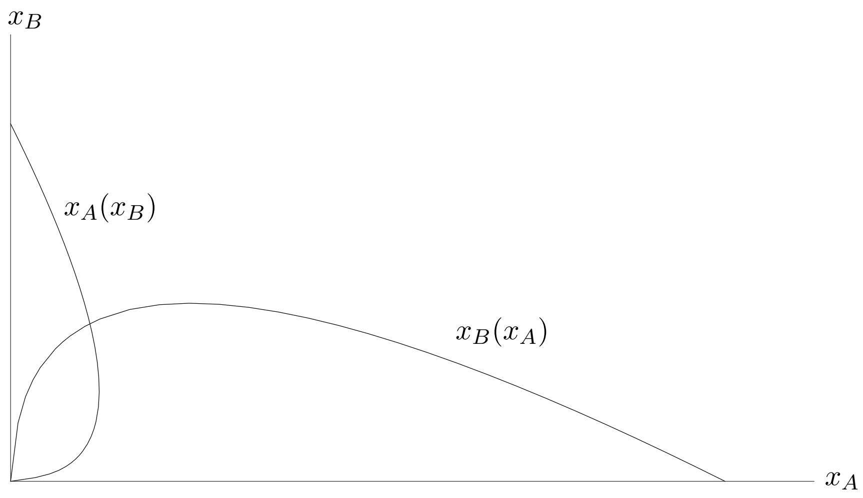

Figure 1: Reaction functions of player A and player B. Player B is the favorite, implying that at the point of intersection her reaction function's slope is positive (strategic complement), whereas the slope of player A's reaction function is negative (strategic substitute).

Equilibrium efforts are monotonically increasing in the own valuation of the prize and are monotonically decreasing (increasing) in the valuation of the other player whenever the other player is the favorite (underdog). This is due to the fact, that the favorite's (underdog's) effort is a strategic substitute (complement) to the underdog's (favorite's) effort - a property that will become important later on (see also figure 1). In our setting the favorite is the player with the higher valuation, because she will spend more effort and hence has a higher probability of winning. Given the equilibrium strategies it is straightforward to compute the equilibrium expected utilities of the players:

$$EU_i^{NE} = \frac{v_i^3}{(v_A + v_B)^2} > 0. \tag{4}$$

This is strictly increasing (decreasing) in the own valuation (valuation of the opponent) and will be our benchmark case. In the following we turn to the analysis of a contests with asymmetric information.

3 Incomplete Information: Bayesian Equilibrium

We model one-sided asymmetric information following Hurley and Shogren (1998b), except, that in our framework players are not heterogeneous with respect to the effectiveness of efforts. Player A's valuation is common knowledge, whereas player B has private information about her valuation v_B. This valuation is drawn from the interval $[\underline{v}_B, \overline{v}_B] \subset \mathbb{R}^+$ according to the distribution function $G(v_B)$, and this is again common knowledge. Therefore the following is also a generalization of the "contest endgame" in Slantchev (2008), who is dealing with a two-point distribution.[5]

We now proceed to analyze the game in more detail. Player A's optimization problem in the incomplete information game (II) is now given by

$$\max_{x_A \in \mathbb{R}_0^+} EU_A^{II} = v_A \, Pr_A[\text{win}] - x_A = v_A \int_{\underline{v}_B}^{\overline{v}_B} \left[\frac{x_A}{x_A + x_B(v_B)} \right] dG(v_B) - x_A. \tag{5}$$

Her first order conditions are given by

$$\frac{\partial EU_A^{II}}{\partial x_A} = v_A \int_{\underline{v}_B}^{\overline{v}_B} \left[\frac{x_B(v_B)}{(x_A + x_B(v_B))^2} \right] dG(v_B) - 1 \geq 0. \tag{6}$$

For player B, who holds full information, the problem is identical to before and therefore her best response is still given by (2). We can use this information to determine the efforts in the Bayesian Nash equilibrium, denoted by the superscript BE.

Proposition 1. *Given the game has an interior solution, in the unique Bayesian Nash equilibrium player A's effort is*

$$x_A^{BE} = \left(\frac{v_A \int_{\underline{v}_B}^{\overline{v}_B} z^{-0.5} \, dG(z)}{v_A \int_{\underline{v}_B}^{\overline{v}_B} z^{-1} \, dG(z) + 1} \right)^2 > 0 \tag{7}$$

and player B's equilibrium effort is given by

$$x_B^{BE} = \sqrt{v_B \, x_A^{BE}} - x_A^{BE}. \tag{8}$$

Proof. See appendix. □

Hurley and Shogren (1998b) already characterized this equilibrium in a somewhat different notation, using the *relative resolve* and *willingness to waste* of the players.[6] However, for our purpose the representation in (7) is more useful.[7]

[5]In section 7 we discuss the connections between Slantchev (2008) and our paper.

[6]Hurley and Shogren (1998b) define willingness to waste as the proportion of the valuation a player is willing to spend in the contest. Relative resolve is a measure of strength of a given player and depends on both valuation of the prize and ability.

[7]Efforts here depend on *negative moments* of the distribution function $G(\cdot)$. Generally, a negative moment of a distribution is given by $E[x^p]$, where $p < 0$. Those moments do not necessarily exist when the support of the density includes zero. We do not encounter this problem since $0 < \underline{v}_B < \overline{v}_B$. Statistical articles dealing with the existence of such moments are for example Chao and Strawderman (1972), Piegorsch and Casella (1985), or Casella and Khuri (2002).

Sometimes it is convenient to use expectation operators instead of integrals, e.g. $x_A^{BE} = \frac{v_A^2 E[\sqrt{1/v_B}]^2}{(v_A E[1/v_B]+1)^2}$.

For an interior solution it has to hold, that all types of player B invest positive effort in equilibrium. Thus, $\sqrt{v_B\, x_A} - x_A \geq 0$, or, equivalently, $v_B \geq x_A$, has to hold. This is trivially fulfilled when player A is the clear underdog, i.e. $v_A \leq \underline{v}_B$. However, since player A will never spend her total valuation of the prize, the contest is noisy, an interior solution can also be found when $\underline{v}_B < v_A$. From (2) we know, that the type of player B with $v_B = \underline{v}_B$ would be the first to run into a corner solution. Therefore, if this type will be active in the contest the game has an interior solution.

Corollary 1. *The game has an interior solution if and only if the following condition is fulfilled:*

$$v_A \leq \tilde{v}_A = \frac{\sqrt{\underline{v}_B}}{\int_{\underline{v}_B}^{\overline{v}_B} \left[\frac{\sqrt{z}-\sqrt{\underline{v}_B}}{z}\right] dG(z)}.$$

Proof. This follows immediately from proposition 1 and (2), given that $v_B = \underline{v}_B$. □

$\tilde{v}_A$ is strictly positive and, as we should expect, increasing in $\underline{v}_B$. The higher $\underline{v}_B$, the less likely it is that $x_A^{BE} > v_B$. If this condition is fulfilled (and only then) all types of player B will spend positive effort and thus we label this an interior solution. There are also scenarios in which only some of the player B types will be active in equilibrium. This will be the case when $v_A > \tilde{v}_A$. Then, some of the player B types will not participate in the contest actively and hence we label this a corner solution. The following proposition characterizes the corner solution equilibria of this game, characterized by the superscript CS.

Proposition 2. *In the unique corner solution Bayesian Nash equilibrium player A's effort is given by*

$$x_A^{CS} = \left(\frac{v_A \int_{\hat{v}_B}^{\overline{v}_B} z^{-0.5} dG(z)}{v_A \int_{\hat{v}_B}^{\overline{v}_B} z^{-1} dG(z) + 1} \right)^2 \qquad (9)$$

where $\hat{v}_B \in (\underline{v}_B, \overline{v}_B]$ is the marginal type of player B implicitly defined by

$$\hat{v}_B = x_A^{CS}.$$

Player B's equilibrium effort is given by

$$x_B^{CS} = \begin{cases} \sqrt{v_B\, x_A^{CS}} - x_A^{CS} & \textit{if} \quad v_B > \hat{v}_B \\ 0 & \textit{else.} \end{cases}$$

Proof. See appendix. □

Player A will always spend strictly positive effort, however, less than in the equilibrium with an interior solution, $x_A^{CS} < x_A^{BE}$. The intuition for this is straightforward: if player B stays passive player A wins the whole prize for sure by spending some $\epsilon > 0$. All types of player B with $v_B \leq \hat{v}_B$ will stay passive in the contest and spend zero effort. All types $v_B > \hat{v}_B$ will invest strictly positive efforts. Corner solutions in Tullock contests are frequently encountered. For example, Grossman and Kim (1995) analyze equilibria of sequential Tullock contests with asymmetric players. In their setting, the leader is the defender of some resource and can invest in defense / fortification to secure her property. Depending on the relative strength of players the leader can fully deter any aggressive behavior. Technically, this is a corner solution, in which only one player invests in the contest. The effect is a similar to ours, however, the cause is a different one, since we analyze a simultaneous game. While in Grossman and Kim (1995) the order of moves might allow the leader to deter any action of the follower, in our setting the lack of information is the driving force, which allows player A to deter some types of player B.

For the remainder of the paper we will assume that we are in an interior solution since this does not change the results qualitatively but reduces complexity.

We are now able to calculate the equilibrium utilities for both players.

Corollary 2. *In the Bayesian equilibrium with interior solution player A's expected utility is*

$$EU_A^{BE} = \frac{v_A^3 \left(\int_{\underline{v}_B}^{\overline{v}_B} z^{-1}\, dG(z)\right) \left(\int_{\underline{v}_B}^{\overline{v}_B} z^{-0.5} dG(z)\right)^2}{\left(v_A \int_{\underline{v}_B}^{\overline{v}_B} z^{-1}\, dG(z) + 1\right)^2} \tag{10}$$

and player B's expected utility is

$$EU_B^{BE} = v_B + x_A^{BE} - 2\sqrt{v_B\, x_A^{BE}} \geq 0.$$

Proof. This follows immediately from (5), (1), and proposition 1. □

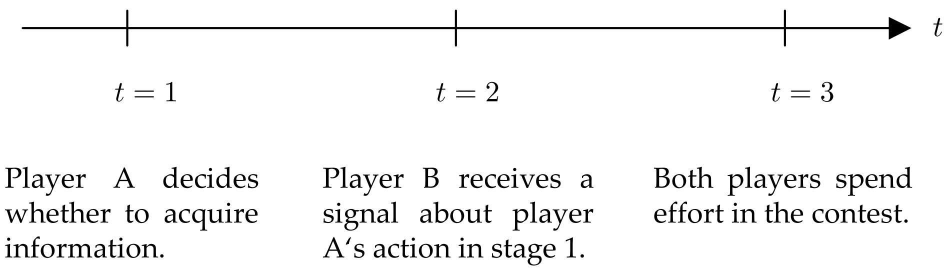

Figure 2: Sequence of moves in the information acquisition game.

4 Acquiring Information

As pointed out in Hurley and Shogren (1998b), one key difference between the complete information contest in section B and the asymmetric information contest just analyzed, is that player A's equilibrium effort is not on her *ex-post* reaction function. Therefore, while ex-ante her effort choice is optimal, this is (most likely) not the case from an ex-post point of view. But does this imply, that player A is necessarily better off under full information? As we will show in the following the answer to this question is no. The reason is that she can commit to a level of efforts which puts her in a position similar to a Stackelberg leader. In this section we will address this issue in more detail.

In the first stage player A decides on whether to acquire information or not. Then, in the second stage, player B receives a signal, which tells her which action player A has chosen before. This signal could be noisy or wrong and is not observable for player A. Conditional on the signal player B forms beliefs. In the third and last stage both players simultaneously choose their efforts in the contest. The sequence of moves can also be seen in figure 2. Let the action of player A in stage 1 be denoted by $\alpha \in \{s, n\}$, and the signal player B observes be given by $\sigma \in \{s, n\}$, where s indicates that player A spied and n indicates that she did not. We allow for a noisy signal. Particularly, the probability to receive the correct signal is

$$Pr[\sigma = i | \alpha = i] \quad = \quad 1 - \epsilon, \quad \epsilon \in [0, 1/2], \; i = s, n. \tag{11}$$

The value of ϵ is common knowledge. We model the signal similar as in the noisy leader game in Bagwell (1995). For $\epsilon = 0$ player B can infer without any doubt what player A chose in stage 1, and thus we call the signal *informative*. However, if $\epsilon = 1/2$ the signal is completely worthless and we call it *uninformative*. For intermediate values of ϵ we say the signal is *noisy*.

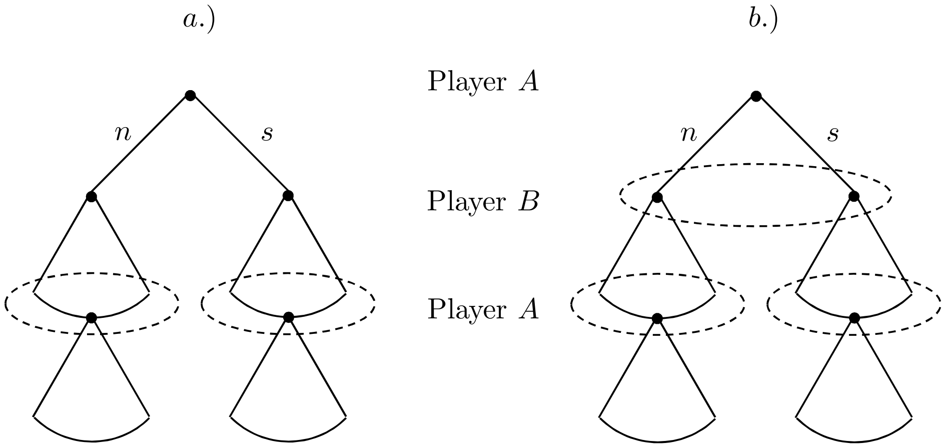

Figure 3: Panel a.) shows the game when $\epsilon = 0$, panel b.) when $\epsilon \in (0, 1/2]$.

A pure[8] strategy for player A is a 3-tuple $\mathcal{A} = \{\alpha, x_A^s, x_A^n\}$ specifying the action α in stage 1 as well as the effort levels x_A^α in the two stage 3 information sets. Player B forms beliefs

$$\beta(\sigma) = (\beta(s), \beta(n))$$

about player A's stage 1 action after receiving the signal $\sigma \in \{s, n\}$ and her strategy specifies her effort in stage 3, $x_B(\sigma) = (x_B(s), x_B(n))$. Together, her belief and strategy form another 2-tuple $\mathcal{B} = \{\beta(\sigma), x_B(\sigma)\}$. The equilibrium concept we employ is perfect Bayesian equilibrium and such an equilibrium is given when the players' strategies are sequentially rational given the beliefs and beliefs are consistent with the strategies.

5 Informative signal: perfectly observable information acquisition

We begin with the analysis of the game when the signal is informative ($\epsilon = 0$). The structure of this game can be seen in panel a.) of figure 3. By deciding over her first stage action α player A effectively chooses which subgame to enter, a subgame with full information or a subgame with one-sided asymmetric information. Acquiring information will induce the full-information subgame, whereas staying ignorant leads to the one-sided asymmetric information subgame. Player A will choose the subgame offering her the higher payoff.

If she chooses to acquire information, player A's expected payoff is the expectation of the

[8]We focus on pure strategy equilibria. Except for the case with intermediate noise we show that this equilibrium is unique.

full-information Nash equilibrium payoff in (4), which is given by

$$E_{v_B}\left[EU_A^{NE}\right] = \int_{\underline{v}_B}^{\overline{v}_B} \left[\frac{v_A^3}{(v_A+z)^2}\right] dG(z). \tag{12}$$

If she abstains from acquiring information in the following one-sided asymmetric information game her equilibrium payoff is given by (10). We define the difference of those two payoffs as

$$\begin{aligned} D &= E_{v_B}\left[EU_A^{NE}\right] - EU_A^{BE} \qquad (13) \\ &= \int_{\underline{v}_B}^{\overline{v}_B} \left[\frac{v_A^3}{(v_A+z)^2}\right] dG(z) - \frac{v_A^3\left(\int_{\underline{v}_B}^{\overline{v}_B} z^{-1}\,dG(z)\right)\left(\int_{\underline{v}_B}^{\overline{v}_B} z^{-0.5}dG(z)\right)^2}{\left(v_A\int_{\underline{v}_B}^{\overline{v}_B} z^{-1}\,dG(z)+1\right)^2} \end{aligned}$$

We assume without loss of generality that player A chooses to acquire information when she is indifferent ($D = 0$). This tie-breaking rule implies $\alpha = s$ whenever $D < 0$. We call $D < 0$ the *value of ignorance condition* (VIC).

When we look more closely at D we see that it depends on different (negative) moments of $G(v_B)$. The following facts show under which circumstances VIC is likely to hold or fail.

Fact 1. *D is U-shaped in $E[1/v_B]$, and has a local minimum at $E[v_A/v_B] = 1$.*

Proof. This follows from inspection of (13). □

Fact 2. *Whenever either $v_A \to 0$ or $v_A \to \infty$, $D > 0$.*

Proof. See appendix. □

Fact 1 shows that VIC is more likely to hold if the expected opponent has a similar valuation as player 1. If we interpret $E[v_A/v_B]$ as a measure of competitive balance in the contest this implies that the closer the contest gets in expectation the more likely it is that VIC holds (ceteris paribus).[9] Fact 2 on the other hand shows that if the contest is very lopsided VIC never holds.

Facts 1 and 2 clearly show that VIC holds more likely if player A's valuation is intermediate relative to $G(v_B)$. We can show that this is generally true if player B's type is drawn from a uniform distribution.

[9]The term "competitive balance" coins the predictability of the outcome of a contest. If the expected ratio of valuations is very high (low) this implies player 1 is likely to be relatively strong (weak). If the expected ratio of valuations is close to unity the expected contest is quite close. See among others Szymanski (2003). Note that competitive balance does not imply that the realized contests must be very close.

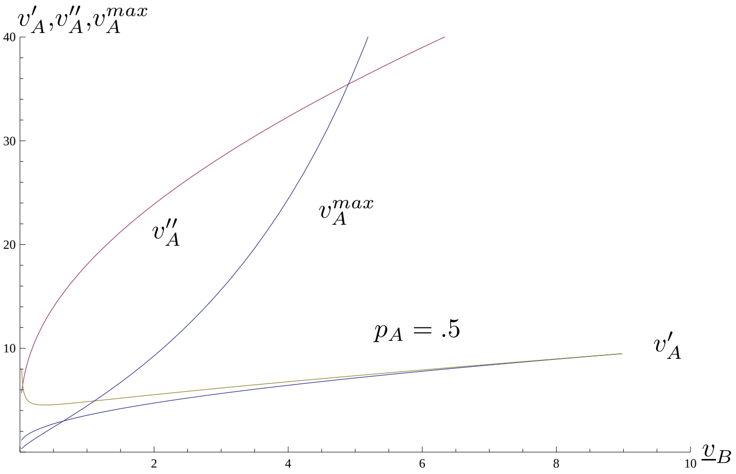

Figure 4: *The lower (v'_A) and upper boundary (v''_A) of values of v_A for which VIC holds, as a function of $\underline{v}_B$. The parabola shows the maximum value of v_A (v_A^{max}) for which we end up in an interior solution. On $p_A = .5$ competitive balance is maximal and equal to one.*

Proposition 3. *If $G(v_B)$ is a uniform distribution on some interval $[\underline{v}_B, \overline{v}_B] \subset \mathbb{R}^+$ there always exists another interval $I = [v'_A, v''_A]$, $v''_A \geq v'_A$, such that $v_A \in I$ implies VIC.*

Proof. See appendix. □

Proposition 3 is a striking result. Whenever player A's beliefs about player B's type are given by a uniform distribution there is an interval of valuations of player A for which VIC holds. Therefore, the value of ignorance is not only given in some curious scenarios, but can be shown to be a quite general result. In figure 4 we plotted the boundaries of the interval I for $\overline{v}_B = 10$ and $\underline{v}_B \in (0, 10)$. Note that here valuations guaranteeing competitive balance $E[v_A/v_B] = 1$ converge to v'_A and hence VIC typically holds for the uniform distribution when A is the expected favorite but not by too much.

We are now able to state the equilibrium of this game.

Proposition 4. *Let $\epsilon = 0$. If $D \geq 0$ the unique perfect Bayesian equilibrium of this game is given by*

$$\mathcal{A}^{\epsilon=0} = \{s, x_A^{NE}(v_B), x_A^{BE}\}$$
$$\mathcal{B}^{\epsilon=0} = \{(\beta(\sigma) = (s, n), x_B(\sigma) = (x_B^{NE}(v_B), x_B^{BE}(v_B))\}.$$

If $D < 0$, the unique perfect Bayesian equilibrium is given by

$$\begin{aligned} \mathcal{A}^{\epsilon=0} &= \{n, x_A^{NE}(v_B), x_A^{BE}\} \\ \mathcal{B}^{\epsilon=0} &= \{(\beta(\sigma) = (s,n), x_B(\sigma) = (x_B^{NE}(v_B), x_B^{BE}(v_B))\}. \end{aligned}$$

What is the intuition for player A being better off staying ignorant in some cases? As we have seen before when we discussed corner solutions, the deterrence effect is quite important. Let us look at this in more detail. First, as pointed out by for example Dixit (1987), in logit or Tullock contests the reaction functions of the different players are not-monotonic and hump shaped. Also, a given player's effort is maximal when her opponent is of the same strength, i.e., the reaction functions of two identical players have a unique intersection at their respective maximum.

Assume there are only two types of player B with respective valuations v_B^h and v_B^l with $v_B^l < v_A < v_B^h$ and that player A is the expected favorite (her probability of success in the Bayesian equilibrium exceeds 50% in expectation). Then player A's equilibrium effort in the Bayesian game is in between her Nash efforts and her effort is maximal in the Nash equilibrium against the high valuation type. So on the one hand she will be able to overcommit effort relative to the Nash equilibrium against the l-type. On the other hand, player A can undercommit relative to the Nash equilibrium against the high valuation type. It is well known since Dixit (1987), that in a Nash equilibrium the favorite's efforts in the contest are strategic substitutes to the underdog's efforts, and that the underdog's efforts are strategic complements to the favorite's efforts.[10] Put differently, if the Nash equilibrium efforts are the reference point, higher efforts of the favorite *decrease* the underdog's efforts, while higher efforts of the underdog *increase* the favorite's efforts. Therefore, it is favorable for her to stay ignorant because this way she can induce lower efforts from both types of player B. Looking at it from another angle, staying ignorant will often bring player A closer to her Stackelberg leader's payoff against both types of opponents solely due to the non-monotonicity of the reaction functions. This intuition carries over to more general cases as well, as we have seen in proposition 3.

Having discussed the intuition it is interesting to ask how general the result is with respect

[10]The terms *strategic substitute* and *strategic complement* are due to Bulow, Geanakoplos, and Klemperer (1985). Generally, the decisions of two or more players in a game are called strategic complements if they mutually reinforce each other. They are called strategic substitutes if they mutually offset one another.

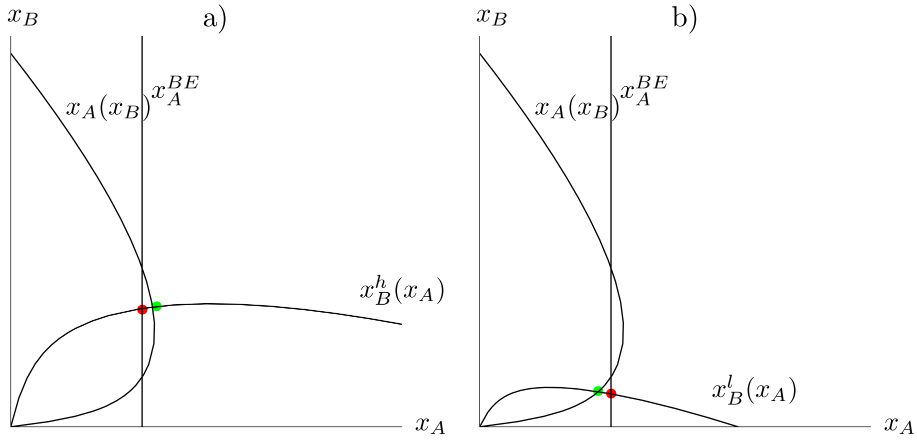

Figure 5: Reaction functions. Panel a) shows the full-information game when player B is of the h-type, panel b) when she is of the l-type. x_A^{BE} denotes in both panels the Bayesian effort of player A. In the Bayesian equilibrium, denoted by the red dot, both types of player 2 spend less effort than in the eauilibrium of the full-information game, denoted by the green dot.

to the contest technology. Varying the discriminatory power of the Tullock contest it is easily shown numerically that the result carries over as long as there are equilibria in pure strategies.[11] In the limit with the discriminatory power equal to infinity the Tullock contest turns into an All-pay auction, where the player with the highest effort wins for sure. Using a result from Kovenock, Morath, and Münster (2009) we can show that in this case VIC never holds.

Proposition 5. *If the contest takes the form of an All-pay auction staying ignorant is weakly dominated for player A and therefore VIC never holds.*

Proof. See appendix. □

This result is important, since it is often assumed Tullock contests and All-pay auctions are similar in many respects. We show there is a qualitative difference in games with asymmetric information when transmission of information is possible. Kovenock, Morath, and Münster (2009) analyze whether players might have an incentive to share their private information in All-pay auctions and find that this is only the case when industry-wide agreements are possible. If the contest is modeled as a Tullock contest instead this finding may change.

[11] The discriminatory power of a contest is a measure of how decisive efforts are. In the case of Tullock contests the discriminatory power is usually modeled as the exponent of the effort choice of any single player, i.e. $Pr_i[win] = x_i^r/(x_A^r + x_B^r)$, $r \geq 0$. If $r = 0$ effort has no effect on determining the winning probability, whereas with $r \to \infty$ the player with the highest effort wins for sure and the contest is an All-pay auction.

Our result that there is a value of ignorance is new to contest theory. We also contribute to a literature establishing a value of ignorance in many circumstances, mostly in the agency literature. For example, Barros (1997) analyzes an oligopolistic industry in which firm owners might have an incentive to stay uninformed about the operations of their agents. The reason is, that because of the information asymmetry the firm commits not to extract the agent's total surplus, what in turn gives the agent incentives to invest effort. But this in turn is also beneficial to the firm. Kessler (1998) analyzes an adverse selection framework in which an agent might have the incentive to acquire information about an economically relevant parameter, like the costs of a given project. She shows, that in equilibrium the agent might voluntarily abstain from gathering this information with positive probability. The intuition is here, that randomizing about whether or not to acquire information he introduces asymmetric information on the part of the principal. This changes the optimal contract in favor of the agent. There are other papers coming to similar findings, that staying ignorant can be beneficial. However, our paper reveals a different channel through which the asymmetry can favor the uninformed player. In the simplest case, being ignorant allows her to use the theoretical chance of facing a strong opponent to gain against the weak, and vice versa.

6 Uninformative or noisy signal

In this section we solve the model when $\epsilon \in (0, 1/2]$ and therefore player A's first stage action is not perfectly observable, i.e. the signal is noisy ($\epsilon \in (0, 1/2)$) or totally uninformative ($\epsilon = 1/2$). The structure of this game can be seen in panel b.) of figure 3. Proposition 6 summarizes the results in this game.

Proposition 6. *If $\epsilon \in (0, 1/2]$ there is a unique perfect Bayesian equilibrium in pure strategies, in which player A spies in stage 1, player B "ignores" the signal in stage B meaning she holds the belief $\beta(\sigma) = (s, s)$, and both players choose equilibrium efforts as in the full-information game (see equation (3)). Therefore, the equilibrium is given by*

$$\begin{aligned} \mathcal{A}^{\epsilon>0} &= \left\{s, x_A^{NE}(v_B), x_A^{NE}(v_B)\right\} \\ \mathcal{B}^{\epsilon>0} &= \left\{\beta(\sigma) = (s, s), x_B^{\epsilon>0}(\sigma) = (x_B^{NE}, x_B^{NE})\right\}. \end{aligned} \tag{14}$$

Moreover, if $\epsilon = 1/2$, this equilibrium is unique.

Proof. See appendix. □

For $\epsilon = 1/2$ the pure-strategy equilibrium is the unique equilibrium of this game. If $\epsilon \in (0, 1/2)$ there might be also equilibria in mixed strategies.

In this scenario both players choose their respective full-information Nash equilibrium strategies. This is due to the fact that player A now cannot credibly use the strategic instrument "ignorance". The noisy signal makes it impossible for her to commit not to deviate from staying ignorant. Therefore we come to similar results as Bagwell (1995) or Morgan and Várdy (2007). Bagwell (1995) shows, that in sequential move strategic form two-player games the strategic advantage of moving first vanishes. The set of equilibria in pure strategies coincides with the set of full information Nash equilibria. Morgan and Várdy (2007) apply a similar framework to a sequential-move Tullock contest where the follower has to pay in order to observe the leaders choice. They find, that there is only a unique pure strategy equilibrium, in which both player chose their respective simultaneous move Nash equilibrium efforts. Our model differs form theirs in two aspects. First, the contest itself is a simultaneous move game. Second, while they assume the follower has the opportunity to learn the leader's stage 1 action, in our model the uninformed player can choose to learn the type of her opponent. Moreover, in our model player A needs a "two-stage commitment". First, depending on the quality of the signal, player A may be able to commit to a particular stage 1 action. Then, if commitment in the first stage is effective, this allows her to commit to some stage 3 action which is favorable for her. The effectiveness of the stage 1 commitment allows her to make another commitment in stage 3.

7 Discussion: Signaling instead of Acquiring Information?

We showed that under certain circumstances it is beneficial for an uninformed player to stay uninformed in a contest. In our model the only way to resolve the informational asymmetry is the uninformed player's information acquisition. However, in a conflict game like a Tullock contest, when one player benefits from ignorance, it might be the case that at least some types of the informed player suffer from this situation. Therefore, it seems intuitive that those types would have an incentive to resolve the informational asymmetry on their own. In principle we could assume many different channels through which such information transmission could happen.

Slantchev (2008) studies a crisis bargaining situation between two players, in which in a first stage player A makes a proposal how to split a given rent. If player B accepts in stage 2, the game ends and both players get the proposed shares. If player B rejects both players compete in a Tullock contest for the rent. Principally, this contest is of two-sided asymmetric information, each player i is with probability p_i strong and otherwise weak. Quite interestingly, in equilibrium the behavior in stages 1 and 2 allows both players to infer at least partly how strong the opponent is, so that in the contest there is either full information or one-sided asymmetric information. Thus, the stage 1 and 2 actions are signals of the players that resolve the informational asymmetry at least partly. However, there are also "feigning" equilibria, in which a strong type demands only a small share in order to provoke a conflict, in which the opponent believes she is weak, therefore spending only low efforts in the contest and thus giving the actually strong player an advantage. For our paper Slantchev (2008) provides two nice insights. First, he shows how rational players behave in a conflict if there are informational asymmetries and there is no opportunity to acquire information but to signal. Second, his model gives us a further application of a one-sided asymmetric information contest. In his model one-sided asymmetric information can emerge endogenously in a bargaining situation, when players are not able to peacefully agree on how to split the desired rent. Our model then explains behavior in the subgame starting after disagreement, if the uninformed player can acquire information himself. Therefore his paper complements ours.

Another recent article studying information transmission is Katsenos (2008). In his paper two players compete for a given rent with common value. Players are heterogeneous in their effort costs and the information structure is two-sided asymmetric information. Specifically, each player is with probability ρ strong, i.e. her costs are low, or with probability $1 - \rho$ weak. Before the contest starts each player has the opportunity to send a signal, which he models as (unproductive) efforts spent already before the contest begins. In the second stage players spend effort in order to win the contest. In this framework he characterizes under which conditions sorting equilibria exist and thus the informational asymmetry can be overcome. He finds that a (symmetric) separating equilibrium, where the strong players signal their type, exists only if the probability of facing a strong opponent ρ is sufficiently low. The intuition here is that signalling strength against a strong opponent will make the competition fiercer and more wasteful. On the other hand a weak opponent will become discouraged by knowing he faces a strong opponent and the contest becomes less wasteful. This finding is interesting in that it shows that the actual holder of the information will only sometimes be able to credibly reveal it, leaving room for our analysis of information acquisition on the part of the uninformed player.

8 Conclusion

In this paper we analyzed Tullock contests with one-sided asymmetric information. First we generally studied the Bayesian equilibria of those games. As in ordinary Tullock contests with full information, players' efforts are strictly increasing in their own valuation of the prize. However, in contrast to standard contests, we have to deal with corner solutions even in simultaneous play. This is due to the fact that the uninformed player sometimes chooses an equilibrium effort larger than the valuation of the informed player. Then we go on showing that even if the uninformed player has the opportunity to (costlessly) acquire the as yet unknown information about the other player's type, she might not have an incentive to do so. The reason is that ignorance is an important strategic instrument in the contest, enabling her to commit to an effort level that is almost surely not on her best response correspondence. Sometimes this effort level will discourage both weak and strong opponents due to the non-monotonicity of reaction functions. When information acquisition is only noisily observable this commitment opportunity is taken away from the uninformed player. In fact, if she could make information acquisition costly for her, she would do so under some circumstances in order to restore her commitment opportunity. Put differently, ignorance often has a value, depending on the distribution of valuations and the quality of the signal. If the contest is an All-pay auction instead of a Tullock contest there is never a value of ignorance. Thus, modeling a contest as an All-pay auction instead of a Tullock contest will lead to unambiguous, and therefore qualitatively different results regarding information acquisition, hence also altering predicted behavior in the contest.

What are the implications of our analysis? First, the probability of conflict is reduced through the asymmetry in information. If we let the assumption of interior solutions in the contest we come to cases where the ignorance of the incumbent might be a means to completely deter some types of entrants. For example, if in the simple example above $c_H = 11$, keeping everything else equal, the high cost firm will never enter. Therefore in contests we should expect to see players that are not too different. Second, due to the preserved asymmetric information in the staying ignorant equilibrium the sum of efforts in the contest might be lower than in the full information case. In settings in which contest efforts are considered to be socially wasteful it might thus be desirable from a policy perspective to preserve the asymmetry (see also Hurley and Shogren (1998a), proposition 3).

A logical extension of our paper would be to allow for more general contest technologies. Our intuition is, that our results qualitatively carry over to all cases where the players's best response functions in the simultaneous-move full information game are hump-shaped, continuous and such that pure-strategy equilibria exist. Another possibility is to extend the analysis towards a n-player contest. Probably the most interesting extension is to analyze the equilibrium information structure in a contest with two-sided asymmetric information, where players have both the opportunity to signal and acquire information. For this purpose our paper as well as the papers of Katsenos (2008) and Slantchev (2008) might be valuable starting points.

Appendix

A Proof of Proposition 1

In order to prove the proposition we first establish the following lemmata:

Lemma 1. *Player A's best-response correspondence in the Bayesian game exists and is single-valued.*

Proof. This follows directly from the strict concavity of EU_A. □

Lemma 2. *Independent of the information structure, player A will in every equilibrium play a pure strategy.*

Proof. To prove this we have to differentiate between four cases.

1. *Player A knows player B's type and player B chooses a pure strategy in equilibrium.* In this case we have a full information game and it is well known that in the unique Nash equilibrium both players choose pure strategies (see Konrad (2009)).

2. *Player A does not know player B's type, and all types of player B play a pure strategy in equilibrium.* In this case we know from lemma 1, that player A has a single best response, and accordingly he chooses a pure strategy in equilibrium.

3. *Player A knows player B's type and player B chooses some mixed strategy $F(x_B)$ in equilibrium.* It is easy to show, that player A's optimization problem in this case is isomorphic to the problem in the Bayesian game. Her expected utility is given by

$$EU_A^{Mix} = v_A \int_{\underline{v}_B}^{\overline{v}_B} \frac{x_A}{x_A + x_B} dF(x_B) - x_A$$

By a simple change of variables ($x_B = x_B(v_B)$ and $F(x_B) = G(v_B)$) we can transform this into the expected utility equation in the Bayesian game, since $x_B(v_B)$ is strictly monotonically increasing in v_B. As before we can use lemma 1 to prove player A plays a pure strategy.

4. *Player A does not know player B's type, and all (some) types of player B play a mixed strategy in equilibrium.* We already saw that player A does not care whether player B randomizes or whether she does not know player B's type. Not knowing the type is strategically similar to a randomization, since in the one case a given player randomizes her pure strategies and in the other case nature randomizes which type chooses a particular pure strategy. If unknown types play a mixed strategy this alters only the distribution of pure strategies chosen by known types, compared to the situation with known types playing a mixed strategy. Therefore, once again we can use lemma 1 to prove that player A will play a pure strategy in equilibrium.

□

Now we are able to prove the proposition. Player A's FOC is given by

$$v_A \int_{\underline{v}_B}^{\overline{v}_B} \left[\frac{x_B(v_B)}{(x_A^{BE} + x_B(v_B))^2} \right] dG(v_B) - 1 \stackrel{!}{=} 0.$$

Using player B's best response function $x_B(x_A) = \sqrt{v_B\, x_A} - x_A$ we can simplify to get

$$\begin{aligned} 1 &= v_A \int_{\underline{v}_B}^{\overline{v}_B} \left[\frac{\sqrt{v_B\, x_A^{BE}} - x_A^{BE}}{v_B\, x_A^{BE}} \right] dG(v_B) \\ \Leftrightarrow 1 &= v_A \int_{\underline{v}_B}^{\overline{v}_B} \left[\frac{1}{\sqrt{v_B\, x_A^{BE}}} - \frac{1}{v_B} \right] dG(v_B). \end{aligned}$$

Using some further algebra we find the equilibrium effort level of player A.

$$
\begin{aligned}
1 &= v_A \int_{\underline{v}_B}^{\overline{v}_B} \left[\frac{1}{\sqrt{v_B\, x_A^{BE}}} - \frac{1}{v_B} \right] dG(v_B) \\
\Leftrightarrow 1 &= v_A \int_{\underline{v}_B}^{\overline{v}_B} \left[\frac{1}{\sqrt{v_B\, x_A^{BE}}} \right] dG(v_B) - v_A \int_{\underline{v}_B}^{\overline{v}_B} \left[\frac{1}{v_B} \right] dG(v_B) \\
\Leftrightarrow x_A^{BE} &= \left(\frac{\int_{\underline{v}_B}^{\overline{v}_B} v_B^{-0.5}\, dG(v_B)}{\int_{\underline{v}_B}^{\overline{v}_B} v_B^{-1}\, dG(v_B) + v_A^{-1}} \right)^2 > 0
\end{aligned}
$$

Player B's equilibrium effort follows trivially from (2). Because the second order conditions for both players are weakly negative for all $x_A, x_B \geq 0$ the efforts are actually optimal choices.

From lemma 2 we know that player A plays a pure strategy in any equilibrium. Thus, we do not need to check for mixed-strategy equilibria. Since player B's best response is also single-valued (see 2) this implies a unique Bayesian Nash equilibrium in pure strategies, provided an equilibrium exists. Therefore the proof is complete. □

B Proof of Proposition 2

If the condition from corollary 1 is not fulfilled, the game has a corner solution. In this equilibrium there is a marginal type B ($\hat{v}_B \in (\underline{v}_B, \overline{v}_B]$) who stays just passive, but all types with a higher valuation $v_B > \hat{v}_B$ will be active. In such an equilibrium player A chooses her effort to maximize

$$
EU_A^c = v_A \left(\int_{\underline{v}_B}^{\hat{v}_B} dG(z) + \int_{\hat{v}_B}^{\overline{v}_B} \frac{x_A}{x_A + x_B(z)} dG(z) \right) - x_A
$$

If she chooses zero effort she gets zero utility. The corresponding FOC is given by

$$
\frac{\partial EU_A^c}{\partial x_A} = v_A \int_{\hat{v}_B}^{\overline{v}_B} \left[\frac{x_B(z)}{(x_A^c + x_B(z))^2} \right] dG(z) - 1 \overset{!}{=} 0.
$$

This is exactly the same as in proposition A except for the different boundaries of integration. Thus, the proof of player A's corner solution effort follows immediately from the proof of proposition 1.

The effort for player B is still given by her best response function in (2). The marginal type of player B stays just passive in equilibrium. Put differently, $\sqrt{\hat{v}_B\, x_A^c} - x_A^c = 0 \Leftrightarrow \hat{v}_B = x_A^c$, since $x_A^c > 0$. Using (9) we get

$$\hat{v}_B = x_A^c = \left(\frac{\int_{\hat{v}_B}^{\overline{v}_B} z^{-0.5} dG(z)}{\int_{\hat{v}_B}^{\overline{v}_B} z^{-1} dG(z) + v_A^{-1}} \right)^2 .$$

The type of player B for whom this equation is fulfilled is the marginal player. According to (2) all types of player B with $v_B > \hat{v}_B$ will spend positive effort $\sqrt{v_B\, x_A^c} - x_A^c$. All other types stay passive and spend zero effort.

As in proposition 1, uniqueness follows from (2), lemma 1, and lemma 2. □

C Proof of Fact 2

We first consider the case $v_A \to 0$. D is given by

$$D = v_A^3 \left(E\left[\frac{1}{(v_A + v_B)^2}\right] - \frac{E\left[v_B^{-1}\right]\, E\left[v_B^{-0.5}\right]^2}{\left(v_A E\left[v_B^{-1}\right] + 1\right)^2} \right),$$

where we change notation from integrals to expectation operators in order to make the exposition clearer. The expectation is always taken with respect to v_B. As $v_A \to 0$, $D \to 0$ as can be seen from the expression for D. To determine from which side D approaches zero we examine the sign of the expression in brackets,

$$E\left[\frac{1}{(v_A + v_B)^2}\right] - \frac{E\left[v_B^{-1}\right]\, E\left[v_B^{-0.5}\right]^2}{\left(v_A E\left[v_B^{-1}\right] + 1\right)^2}.$$

Let us set $v_A = 0$ to determine the sign of the expression in brackets.

$$E\left[\frac{1}{v_B^2}\right] - E\left[\frac{1}{v_B}\right]\, E\left[\frac{1}{\sqrt{v_B}}\right]^2$$

We use Jensen's inequality twice to determine the sign:

$$\begin{aligned}
E\left[\frac{1}{v_B^2}\right] - E\left[\frac{1}{v_B}\right]\, E\left[\frac{1}{\sqrt{v_B}}\right]^2 &> E\left[\frac{1}{v_B^2}\right] - E\left[\frac{1}{v_B}\right]\, E\left[\frac{1}{v_B}\right] \\
&> E\left[\frac{1}{v_B^2}\right] - E\left[\frac{1}{v_B}\right]^2 \\
&> E\left[\frac{1}{v_B^2}\right] - E\left[\frac{1}{v_B^2}\right] = 0.
\end{aligned}$$

As we can see D is positive when $v_A \to 0$.

Now we turn to $v_A \to \infty$. Again we only need to look at the expression in brackets. If we take the limit to infinity only the highest order terms of v_A have to be considered. Hence our expression reduces to:

$$\frac{1}{v_A^2} - \frac{E\left[\frac{1}{v_B}\right] E\left[\frac{1}{\sqrt{v_B}}\right]^2}{v_A^2 E\left[\frac{1}{v_B}\right]^2} = \frac{1}{v_A^2} - \frac{E\left[\frac{1}{\sqrt{v_B}}\right]^2}{v_A^2 E\left[\frac{1}{v_B}\right]} = \frac{1}{v_A^2}\left(1 - \frac{E\left[\frac{1}{\sqrt{v_B}}\right]^2}{E\left[\frac{1}{v_B}\right]}\right).$$

Again we make use of Jensen's inequality:

$$1 - \frac{E[\frac{1}{\sqrt{v_B}}]^2}{E[\frac{1}{v_B}]} > 1 - \frac{E[\frac{1}{v_B}]}{E[\frac{1}{v_B}]} = 0.$$

We see, that $D > 0$ when $v_A \to \infty$. This completes the proof. □

D Proof of Proposition 3

Let $G(v_B)$ be a uniform distribution on $[\underline{v}_B, \overline{v}_B] \subset \mathbb{R}^+$. Using Corollary 2 we can calculate the difference in utilities $D = E_{v_B}\left[EU_A^{NE}\right] - EU_A^{BE}$, where the expected utility from information acquisition is

$$E_{v_B}\left[EU_A^{NE}\right] = \frac{v_A^3}{v_A + \underline{v}_B} - \frac{v_A^3}{v_A + \overline{v}_B}$$

and the expected utility from ignorance is

$$EU_A^{BE} = \frac{4v_A^3\left(\sqrt{\overline{v}_B} - \sqrt{\underline{v}_B}\right)^2(\ln[\overline{v}_B] - \ln[\underline{v}_B])}{(v_A(\ln[\overline{v}_B] - \ln[\underline{v}_B]) + \overline{v}_B - \underline{v}_B)^2}.$$

Minor transformations show that player A is better of staying ignorant, when her valuation is intermediate, $v'_A < v_A < v''_A$, where the lower and upper bound are given by

$$v'_A = \frac{\left(\sqrt{\overline{v}_B} - \sqrt{\underline{v}_B}\right)^3 - \left(\ln\left[\frac{\overline{v}_B}{\underline{v}_B}\right]\right)^{-\frac{1}{2}} 2\Psi}{\left(4\sqrt{\underline{v}_B} - 4\sqrt{\overline{v}_B} + \left(\sqrt{\underline{v}_B} + \sqrt{\overline{v}_B}\right)\ln\left[\frac{\overline{v}_B}{\underline{v}_B}\right]\right)},$$

and

$$v''_A = \frac{\left(\sqrt{\overline{v}_B} - \sqrt{\underline{v}_B}\right)^3 + \left(\ln\left[\frac{\overline{v}_B}{\underline{v}_B}\right]\right)^{-\frac{1}{2}} 2\Psi}{\left(4\sqrt{\underline{v}_B} - 4\sqrt{\overline{v}_B} + \left(\sqrt{\underline{v}_B} + \sqrt{\overline{v}_B}\right)\ln\left[\frac{\overline{v}_B}{\underline{v}_B}\right]\right)},$$

where

$$\Psi = \sqrt{(\overline{v}_B - \underline{v}_B)\left(\underline{v}_B - \overline{v}_B + \sqrt{\underline{v}_B\overline{v}_B}\ln\left[\frac{\overline{v}_B}{\underline{v}_B}\right]\right)^2}.$$

From inspection of v''_A we see that this boundary is positive whenever the denominator is positive, i.e. $4\sqrt{\underline{v}_B} - 4\sqrt{\overline{v}_B} + \left(\sqrt{\underline{v}_B} + \sqrt{\overline{v}_B}\right)\ln\left[\frac{\overline{v}_B}{\underline{v}_B}\right] > 0$. This is always the case when $\overline{v}_B > \underline{v}_B$, as can be shown by simple manipulations (available upon request). □

E Proof of Proposition 4

The proof that this is indeed an equilibrium follows from the discussion in the text. Uniqueness follows from uniqueness of the equilibria in the subgames following the first stage decision and the assumption that for $D = 0$ player A will choose to acquire information. Hence, player A will never mix between s and n in stage 1. □

F Proof of Proposition 5

The proof partly follows from the analysis of Kovenock, Morath, and Münster (2009). Proposition 1 and 2 in their paper imply that player A's expected utility from acquiring information is strictly larger than her payoff in the one-sided asymmetric information game for all $v_A \in (\underline{v}_B, \overline{v}_B]$. Hence player A will always prefer to acquire information about player B. If $v_A \leq \underline{v}_B$ in any full-information Nash equilibrium (Hillman and Riley (1989)) and in the one-sided asymmetric information Bayesian equilibrium player A's expected utility will be zero. Therefore also in this case VIC does not hold. Lastly for $v_A > \overline{v}_B$ the expected full information Nash equilibrium payoff is equal to $\int_{\underline{v}_B}^{\overline{v}_B}(v_A - v_B)dG(v_B)$. Proposition 2 of Kovenock, Morath, and Münster (2009) is generalizable to show this payoff is strictly larger than the payoff with one-sided asymmetric information also for $v_A > \overline{v}_B$. □

G Proof of Proposition 6

Given $A = \left\{s, x_A^{NE}\right\}$, by consistency of beliefs player B will always believe that player A spies regardless of her signal and it is optimal for her to play x_B^{NE}. On the other hand, given $B = \left\{\beta(\sigma) = (s, s), x_B(\sigma) = (x_B^{NE}, x_B^{NE})\right\}$ player A will find it optimal to acquire information and play her best response x_A^{NE}. We prove uniqueness (in pure strategies) by contradiction. Assume that player A does not acquire information. By consistency of beliefs player B will believe that player A did not acquire information regardless of her signal. Hence she will play x_B^{BE}. In this case player A will find it profitable to deviate and acquire information as player B will not spot the deviation and react on it. $\square$

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