

André PREUMONT  
*Université Libre de Bruxelles*  
*Active Structures Laboratory*

Mechatronics: Dynamics  
of Electromechanical and  
Piezoelectric Systems

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*”Tenez, mon ami, si vous y pensez bien,  
vous trouverez qu’en tout,  
notre véritable sentiment n’est pas celui  
dans lequel nous n’avons jamais vacillé;  
mais celui auquel nous sommes le plus  
habituellement revenus.”*

Diderot,  
(*Entretien entre D’Alembert et Diderot*)



## Preface

The objective of my previous book, *Vibration Control of Active Structures*, was to cross the bridge between Structural Dynamics and Automatic Control. To insist on important control-structure interaction issues, the book often relied on “ad-hoc” models and intuition (e.g. a thermal analogy for piezoelectric loads), and was seriously lacking in accuracy and depth on transduction and energy conversion mechanisms which are essential in active structures. The present book project was initiated in preparation for a new edition, with the intention of redressing the imbalance, by including a more serious treatment of the subject. As the work developed, it appeared that this topic was broad enough to justify a book on its own.

This short book attempts to offer a systematic and unified way of analyzing electromechanical and piezoelectric systems, following a Hamilton-Lagrange formulation. The transduction mechanisms and the Hamilton-Lagrange analysis of classical electromechanical systems have been addressed in a few excellent textbooks (e.g. *Dynamics of Mechanical and Electromechanical Systems* by Crandall et al. in 1968), but to the author’s knowledge, there has been no similar systematic treatment of piezoelectric systems.

The first three chapters are devoted to the analysis of mechanical systems, electrical networks and classical electromechanical systems, respectively; Hamilton’s principle is extended to electromechanical systems following two dual formulations. Except for a few examples, this part of the book closely follows the existing literature. The last three chapters are devoted to piezoelectric systems. Chapter 4 analyzes discrete piezoelectric transducers and their introduction into a structure; the approach parallels that of the previous chapter with the appropriate energy and coenergy functions. Chapter 5 analyzes distributed systems, and focuses on piezoelectric beams and laminates, with particular attention to the way the piezoelectric layers interact with the supporting structure (piezoelectric loads, modal filters, etc...). Chapter 6 examines energy conversion from the perspective of active and passive damping; a unified approach is proposed, leading to a meaningful comparison of various active and passive techniques, and design guidelines for maximizing energy conversion.

This book is intended for mechanical engineers (researchers and graduate students) who wish to get some training in electromechanical and piezoelectric transducers, and improve their understanding of the subtle interplay between mechanical response and electrical boundary condi-

tions, and *vice versa*. In so doing, we follow the famous advice given by Prof. Joseph Henry to Alexander Graham Bell, who had consulted him in connection with his telephone experiments in 1875, and lamented over his lack of the electrical knowledge needed to overcome his mechanical difficulties. Henry simply replied: “*Get it*”. The beauty of the Hamilton-Lagrange formulation is that, once the appropriate energy and coenergy functions are used, all the electromagnetic forces (electrostatic, Lorentz, reluctance forces,...) and the multi-physics constitutive equations are automatically accounted for.

## Acknowledgements

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## Notation

Notation is always a source of problems when writing a book, and the difficulty is further magnified as one attempts to address interdisciplinary subjects, which blend disconnected fields with a long history, each with its own, well established notation. This book is no exception to this rule, since mechatronics mixes, analytical mechanics, structural mechanics, electrical networks, electromagnetism, piezoelectricity and automatic control, etc.

The notation has been chosen according to the following rules: *(i)* We shall follow the *IEEE Standard on Piezoelectricity* as much as we can. *(ii)* When there is no ambiguity, we will not make explicit distinction between scalars, vectors and matrices; the meaning will be clear from the context. In some circumstances, when the distinction is felt necessary, column vectors will be made explicit by  $\{ \ }$  (e.g.  $\{T\}$  will denote the stress

vector, while  $T_{ij}$  denotes the stress tensor). (iii) The partial derivative will be denoted either by  $\partial/\partial x_i$  or by the subscript  $_{,i}$  (the index after the comma indicates the variable with respect to which the partial derivative is taken); the choice of one notation or the other will be guided by clarity, compactness and conformity to the classical literature. Similarly, summation on repeated indexes (Einstein's summation convention) will be assumed even when it is not explicitly mentioned.

André Preumont  
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