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TRANSPORTATION MODE CHOICE

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Summary

This paper presents a dynamic model of transportation mode choice and evolution of public transportation service based on some simple assumptions of individual behavior and economic necessities for providing transportation service. Critical values are shown to exist for the fares charged, for the cost of providing service, for the demand and supply of transportation (and for other parameters) at which the system will bifurcate to different possible states of the system; critical thresholds must be reached in the quality of the network to observe its growth. Also shown is the role of history and the role that fluctuations in individual behavior and mode strategy play in the way the system structures, that is, in the evolution of the relative number of users of each mode and in the level of service obtained.

I. Introduction

In a previous paper (Deneubourg, de Palma and Kahn, 1979),[1] we developed a model of transportation mode choice of two variables, the number of people who choose the automobile and the number of people who choose public transportation. That model showed clearly how the system structures in response to individual behavior and showed the role that fluctuations play on the response of the system when different mode choices are made. However, the model did not take account of transportation costs and service.

In this paper, we extend the previous model by taking into account the service offered by the public transportation mode which is assumed to be a function of the number of users of the mode, of the fares charged by the mode, and of the costs of offering the service. By so doing, we shall find here the occurrence of an optimum level of service which is related to the fares charged. We also find the occurrence of critical values for the fares, for the cost of maintaining a level of service, for the demand for transportation, and for the publicity and strength of imitative behavior which will be chosen as bifurcation parameters for the evolution of the system.

As in the previous paper, we make no pretense at developing a model of transportation mode choice which captures all the decisions which go into such mode choices. The intention, rather, is to show how some decisions by individuals and by the public transportation company interact in a dynamic system and how fluctuations can lead to different evolutionary paths.

We may point out that a parallel approach has been developed by A. G. Wilson in the framework of catastrophe theory (Wilson, 1979).[2]

II. The Model

As in reference 1, we make certain assumptions on individual behavior (or, more precisely, on the average behavior of a group of individuals) and through the nonlinearities in this behavior we shall find how the system structures when thresholds for bifurcation are reached.

The system consists of three variables, the number of automobile users, x, the number of public transportation users, y, and the public transportation mode characterized by the global level of service it offers, L (this could be the number of buses, for example). We assume that an important determinant for individuals to choose the automobile is its speed (in a forthcoming paper we shall also take into account the convenience of the automobile expressed by its ubiquitous connectivity). We also consider that imitative behavior (or, in general, any behavior by which the presence of users increases the number of users) plays a role for such mode choice. Both determinants, the speed and the imitative behavior are functions of x, themselves.

For the public transportation mode which, to be specific, is taken to be the bus (though we could take the subway, with however a different scale for the parameters), we assume that the service offered and fares charged affect its usage, imitative behavior plays a role, and as a policy variable, we also assume bus usage may be affected by publicity or advertisement.

We express the above assumptions on mode choice in the dynamical equations for the evolution of x, y and L, of which the first two are given by (see Reference 1, Equation 6):

$$\dot{x} = \frac{D A_x}{A_x + A_y} - x \qquad \dot{y} = \frac{D A_y}{A_x + A_y} - y \qquad (1)$$

where $\dot{x}$, $\dot{y}$ are the time rates of change of x and y, respectively. D is the demand for transportation which is assumed to be so slowly varying compared to the time variation of x and y that it may be considered as constant. That is to say, no new users are brought into the system during the time of interest. A_x is the attractivity for the automobile which we take to be its speed. We include the imitative behavior of people in this term, as well. A_y is the attractivity for the bus which will involve the service offered by the bus mode, the fares charged, the amount of advertisement for bus usage and also the imitative behavior of individuals.

The rationale of these dynamical equations is fully explained in Reference 1, and we only note here that $\dot{x} + \dot{y} = D - (x + y)$, so that in the steady state we have $D = x + y$.

The third equation to complete our system is for the evolution of bus service and is assumed to be given by:

$$\dot{L} = \nu y - KL \qquad (2)$$

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where v is the fare charged (and thus vy is the revenue received) and K is the maintenance cost per unit of service offered. The equation simply states that bus service will grow in time if revenues exceed the cost of providing service.

It now remains to give explicit representations to the attractivities, A_x and A_y in Equation (1). For the automobile attractivity function, A_x, we assume as in Reference 1:

$$A_x = v_x \alpha_1 x \tag{3}$$

where v_x is the automobile speed and α_1 measures the strength of the imitative term $\alpha_1 x$. Also, as in Reference 1, we take the speed to be an inverse function of x (congestion effect see Haight, 1963;[3] R. Herman and I. Prigogine, 1979),[4] and neglect the traffic interaction between cars and buses:

$$V_x = \frac{1}{a + bx} \tag{4}$$

where a and b are positive constants.

For the bus attractivity, A_y, we take the form

$$A_y = \frac{L}{v^2}\left(\theta + \alpha_2 y\right) \tag{5}$$

which states that the attractivity is proportional to the service offered, L, the publicity or information, θ (assumed positive) and the importance of imitative behavior measured by $\alpha_2 y$, and inversely proportional to the second power of the fares, v, charged. We note that the form used for the dependence of A_y on the various parameters will affect the structure of the system (because of the dependence of the structure on the non-linearities). However, as it is not our intention here to reproduce an experimental result, and only to show how the system structures when non-linearities are present, we present these non-linearities (dependence of A_y on the parameters) as only reasonable possibilities. Though, we should point out that it is not difficult to alter these dependencies when sufficiently valid data justifies this.

If we further simplify the problem as in Reference 1 by taking a = 0, b = 1 in Equation (4) which is equivalent to assuming a constant attractivity for the car, we obtain as our system of dynamical equations

$$\dot{x} = \frac{D\alpha_1}{\alpha_1 + \frac{L}{v^2}(\theta + \alpha_2 y)} - x$$
$$\dot{y} = \frac{DL/v^2\,(\theta + \alpha_2 y)}{\alpha_1 + \frac{L}{v^2}(\theta + \alpha_2 y)} - y\ ; \quad \dot{L} = vy - KL \tag{6}$$

This system may be solved analytically. One stationary state ($\dot{x} = 0$, $\dot{y} = 0$, $\dot{L} = 0$) of the system is

$$x = D,\ y = 0,\ L = 0 \tag{7}$$

which states that the total demand for transportation is provided by the automobile.

When we perform a stability analysis, subjecting system (6) to perturbations δx, δy, δL, we find that the stationary state given by (7) is stable only when the costs, K, for providing bus service, are above a certain critical value K_c

$$K > K_c = \frac{D\theta}{v\alpha_1} \tag{8}$$

since then there is no incentive for instituting bus service; we find similar relationships for the other parameters (see the figures).

In addition to the stationary state given by Equation (7), we find two other possible stationary states of the system (6), given by

$$y^{\pm} = \frac{1}{2}\left(D - \frac{\theta}{\alpha_2}\right) \pm \frac{1}{2}\sqrt{\left(D + \frac{\theta}{\alpha_2}\right)^2 - 4\frac{\alpha_1 v K}{\alpha_2}}$$
$$x^{\mp} = \frac{1}{2}\left(D + \frac{\theta}{\alpha_2}\right) \mp \frac{1}{2}\sqrt{\left(D + \frac{\theta}{\alpha_2}\right)^2 - 4\frac{\alpha_1 v K}{\alpha_2}} \tag{9}$$
$$L^{\pm} = v y^{\pm}/K$$

The stability analysis shows that when these solutions are real positive, the (x^-, y^+, L^+) solution is stable and the (x^+, y^-, L^-) solution is unstable. The implications of this for causing transitions between stable states will be seen in the next section when we discuss the results of a numerical example.

The solution (9) will be real (positive or negative) if the cost K for providing bus service is below a critical value K^c:

$$K < K^c = \frac{\alpha_2 (D + \theta/\alpha_2)^2}{4\alpha_1 v} \tag{10}$$

Both solutions will exist physically if $K > K_c = \frac{D\theta}{\alpha_1 v}$ (see Equation (8) and $D > \theta/\alpha_2$. If $K < K_c$ with $D > \frac{\theta}{\alpha_2}$ only the stable solution ($x^- y^+$, L^+) will be positive (will exist physically). This is illustrated in the schematic.

III. Discussion of a Numerical Example

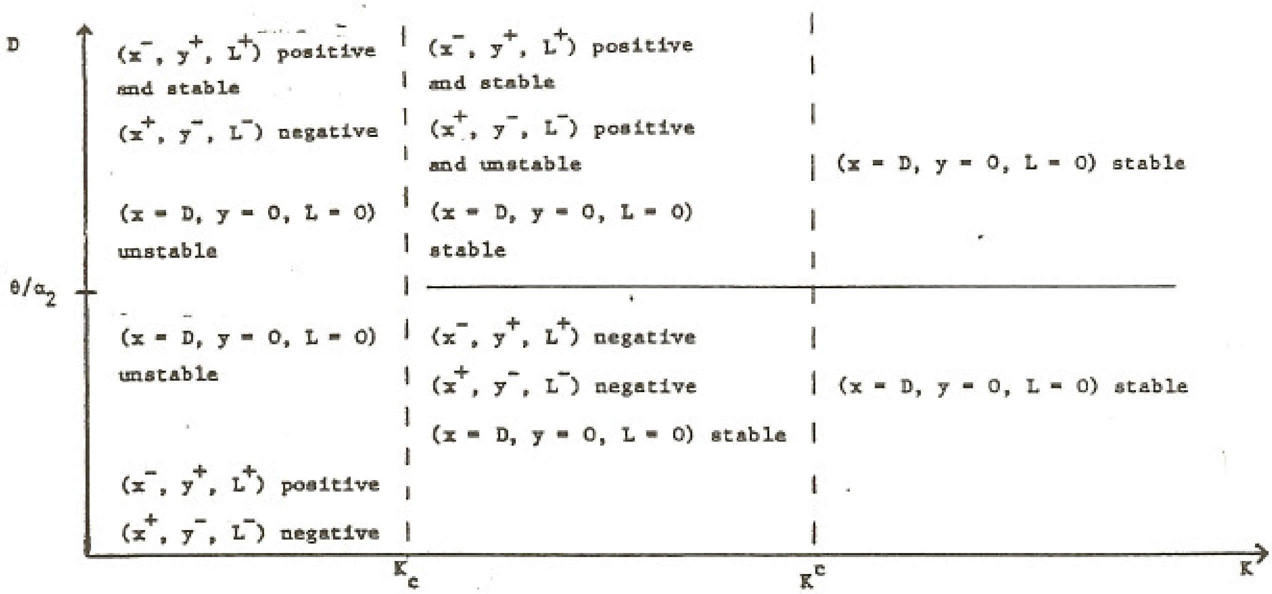
In this section we discuss the structure of the system as a function of the parameters of the system. The numerical values of the parameters are given in the figures.

A. Fares

Figure 1 shows the three variables of the system, the number of people choosing the car mode, x, the number choosing the bus, y, and the service offered by the bus mode, L, as a function of increasing fares v charged by the bus company.

As fares increase but remain below a critical value only one stable stationary state is possible, the (x^-, y^+, L^+) state. The other stationary state (x = D, y = 0, L = 0) exists but is unstable in this low fare regime, so that any perturbation from this state, no matter how slight, will cause the system to jump to the stable state (and we note that in the real world perturbations always are occurring). In this regime, the number of bus users decrease (because of the increasing fares), the bus service improves (because of increasing revenues) and the number of automobile users increases (corresponsing to the decrease in the number of bus users).

As fares continue to increase beyond v_c, but still remain below v^c, two stable stationary states exist and one unstable stationary state exists in the system. In the stable (x^-, y^+, L^+) state the number of bus users continues to decline with increasing fares, and there is



SCHEMATIC SHOWING DIFFERENT REGIONS FOR STABILITY AND SIGN OF ROOTS

a corresponding increase in the number of car users. Bus service continues to improve with increasing fares but then reaches a maximum and begins to rapidly deteriorate because the increasing fares being charged cannot make up for the resultant loss of passengers.

The optimum fare ν_m that should be charged for the best service may be computed analytically. This is most easily done by first finding the optimum number of passengers for producing maximum service. This is obtained from

$$\frac{\partial L}{\partial y} = \frac{\nu}{K} + \frac{y}{K}\frac{\partial \nu}{\partial y} \tag{11}$$

We obtain for the optimum number y_m

$$y_m = \frac{1}{3}\left[D - \frac{\theta}{\alpha_2} + \sqrt{\left(D - \frac{\theta}{\alpha_2}\right)^2 + \frac{3D\theta}{\alpha_2}}\right] \tag{12}$$

The optimum fare ν_m is then given by

$$\nu_m = \frac{\alpha_2}{\alpha_1 K}\left[\frac{D\theta}{\alpha_2} + y_m\left(D - \frac{\theta}{\alpha_2}\right) - y_m^2\right] \tag{13}$$

where we have used Equation (9) to obtain ν as a function of y. We also note that (11) may be put into the form $\frac{dy}{y} \Big/ \frac{d\nu}{\nu} = -1$ which shows that the maximum L is achieved when the elasticity is -1.

We now point out the consequences of this kind of structure of two stable stationary states and one unstable state on the response of the system to fluctuations..

As the fares continue to increase in this regime $\nu_c < \nu < \nu^c$, it becomes more and more likely that a fluctuation in the number of car or of bus users will be found that will cause the system to jump to the zero bus users state.

Finially, for still higher fares ν exceeding the critical value ν^c, the system becomes insensitive to perturbations, adopting the $(x = D, y = 0, L = 0)$ stationary state which is stable in this regime of high fares $\nu > \nu^c$.

B. Cost of Providing Service

Figure 2 shows the bifurcation diagram as a function of costs of providing services K.

When the costs are below the critical value K_c, there is one stable stationary state in which buses and cars co-exist. The all-car solution is unstable in this regime of low costs.

As might be expected, the level of bus service decreases with increasing costs, and hence the number of bus users decreases with a corresponding increase in car users.

As costs continue to rise, in the range $K_c < K < K^c$, the system can exist in one of two possible stable stationary states. If the system is in the (x^-, y^+, L^+) state, the chances of remaining there diminish with increasing costs, as the strength of a perturbation which could cause the system to jump to the $(x = D, y = 0, L = 0)$ state decreases with increasing costs.

When costs exceed the critical value K^c, no one chooses the bus and only the $(x = D, y = 0, L = 0)$ stationary state is stable.

C. Publicity for the Bus

Figure 3 shows the relationship between x y and L and the amount of publicity or advertisement for bus usage.

The system has one stable stationary state below a critical value θ^c, two stable stationary states in the range $\theta^c < \theta < \theta_c$ and one stable stationary state for θ beyond θ_c. The figure points to the need to exceed a critical value θ^c before people will choose the bus mode, but once this critical value is exceeded, further publicity has very little effect on increasing ridership.

However, if the system happens to be in the all-car state, increasing publicity does have a strong effect on increasing the likelihood that a fluctuation will cause the system to change to the mixed mode state. And when $\theta > \theta_c$ any perturbation will cause the system to jump to the mixed mode state.

We remark here that once the publicity exceeds θ_c (so that any perturbation will cause the system to jump to the bus users state), it is not necessary to maintain the same high level of publicity for the system to remain in this state, as is evident from the figure. This is an example of the phenomenon of hysteresis.

D. Demand for Transportation

Figure 4 shows the effect of total demand for transportation D on mode choice x and y and on bus service, L.

When there is an insufficient total demand for transportation, no bus service is offered. Not until a critical value of demand D^c is exceeded is bus service offered. The zero bus state, however, is still a possible stable stationary state in the range of demand $D^c < D < D_c$ but becomes increasingly less likely because smaller fluctuations can cause the system to jump to the mixed mode solution.

For still higher demand, $D > D_c$, the zero bus users state becomes unstable. In the stable stationary state, the number of car users declines with increasing demand as congestion effects become more pronounced, the number of bus users increases as people leave their cars, and bus service improves as more revenues are received because of the increased bus usage.

IV. CONCLUSIONS

In this paper we have presented a dynamic model of transportation mode choice in which mode choice was based on individual behavioral characteristics and on the service offered by the mode.

In examining the stationary states of the system and the stability of these states to fluctuations, we found the existence of critical values of the parameters (fares charged, cost of providing service, demand for transportation, etc.) at which the system bifurcated to a new solution.

For some range of the parameters we found that only one of two possible states of the system was stable to fluctuations and hence in this range the system would adopt one of the two possible states (the stable one).

In another range of values of the parameters we found the existence of two stable states separated by an unstable one. This kind of structure, in which two stable states exist, points to the role of history (through the initial states and through fluctuations) in determining which state the system will adopt (since either one is theoretically possible).

Further, this kind of structure also points to the importance of fluctuations in influencing the behavior of a system as sufficiently strong fluctuations can cause the system to jump from one stable state to another. The size of the fluctuation needed depended on the closeness of the unstable state to one of the stable ones which, in turn, depended upon the values of the parameters of the system.

We point out that the concept of self-organization which appears under certain conditions involving the feedback between a system and its environment, springs from the work done in non-linear thermodynamics (Nicolis, Prigogine 1977)[5] and has found specific applications in biology, ecology and the social sciences.

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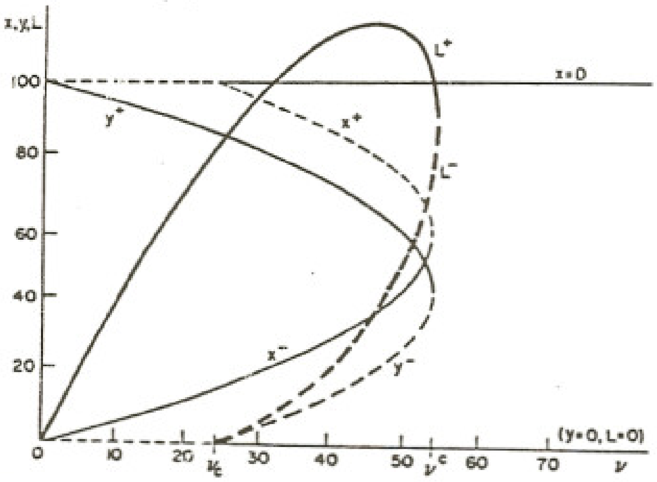

Figure 1 - MODE CHOICE AND QUALITY OF SERVICE VERSUS FARES

Solid lines: stable stationary state
Dashed lines: unstable stationary state
Parameter values: $\alpha_1 = 5$, $\alpha_2 = 2$, $\theta = 30$, $K = 25$, $D = 100$

Critical values: $\nu_c = 24$, $\nu^c = 52.9$

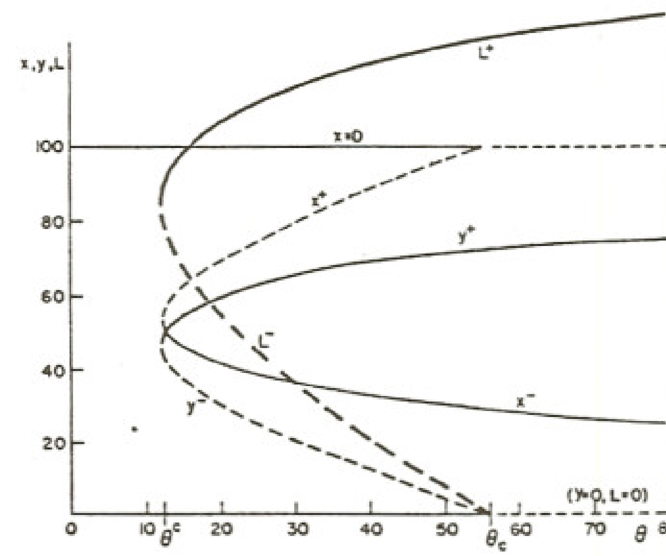

Figure 3 - MODE CHOICE AND QUALITY OF SERVICE VERSUS PUBLICITY

Solid lines: stable stationary state
Dashed lines: unstable stationary state
Parameter values: $\alpha_1 = 5$, $\alpha_2 = 2$, $\nu = 45$, $K = 25$, $D = 100$

Critical values: $\theta_c = 56.3$

$\theta^c = 12.1$

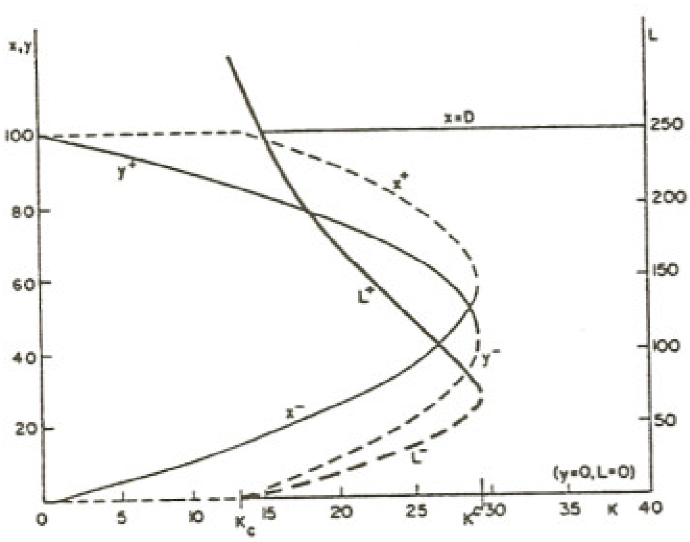

Figure 2 - MODE CHOICE AND QUALITY OF SERVICE VERSUS COSTS

Solid lines: stable stationary state
Dashed lines: unstable stationary state
Parameter values: $\alpha_1 = 5$, $\alpha_2 = 2$, $\theta = 30$, $\nu = 45$, $D = 100$

Critical values: $K_c = 13.3$, $K^c = 29.4$

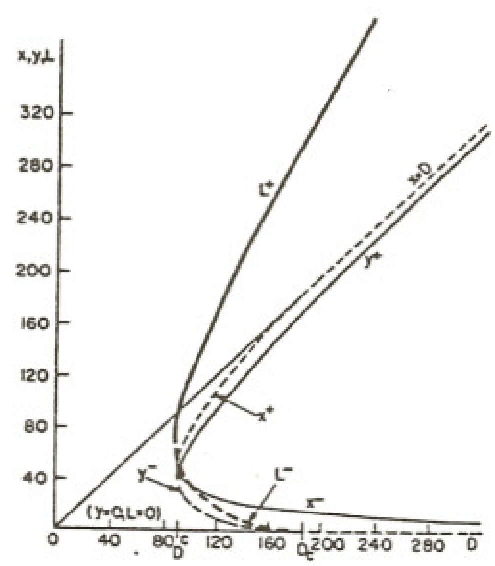

Figure 4 - MODE CHOICE AND QUALITY OF SERVICE VERSUS TOTAL DEMAND FOR TRANSPORTATION

Solid lines: stable stationary state
Dashed lines: unstable stationary state
Parameter values: $\alpha_1 = 5$, $\alpha_2 = 2$, $\nu = 45$, $K = 25$, $\theta = 30$

Critical values: $D_c = 187.5$, $D^c = 91.1$