Infinite curves on closed surfaces

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Then work arose from studying flows (continuous 1-parameter transformation groups) on surfaces. Surfaces considered are those of nonpositive Euler characteristic χ, since for those with $\chi > 0$ the well-known Poincaré-Bendixson theory provides a complete description of possible qualitative types of behavior of trajectories. A new feature in case $\chi \leq 0$ is that the universal covering surface is noncompact and can be endowed by a structure of Euclidean or hyperbolic (Lobachevskiy) plane; thus one can lift the trajectory to this plane and ask about behavior of the lifted trajectory, so to say, at infinity, using geometric notions appropriate to the above mentioned structure. Such approach in rather particular cases was suggested by A.Weil in 30-s, then forgotten and revived by N.Markley (USA) and myself in 60-s.

In the course of research the subject of the theory of ordinary differential equations (ODE) and dynamical systems was somewhat extended in a natural way and now my work concerns geometrical questions which can be asked not only about the (semi)trajectory, but about any (semi)infinite continuous curve having no selfintersections (again lifted to the covering plane). Special attention is paid to the "intermediate" case of leaves of foliations on surfaces with only finite number of singularities. So, I consider several classes of curves and compare these classes with respect to several properties. The latter are such that they do not change if the lifted curve is replaced by another curve lying at a bounded Frechet distance from the first curve. (E.g., such is the property that the lifted curve is (or is not) bounded.) Such properties are, in a sense, more "elementary", "primitive" than properties of the limit behaviour of the trajectories usually considered in the qualitative theory of ODE.