

Basics of Open String Field Theory

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Abstract

We give an elementary introduction to Open String Field Theory and the physics of Tachyon Condensation in the context of the Bosonic String

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1 Introduction

String Theory is, at the time of writing, a very (if not the only) promising way to describe our universe in a consistent and unified theoretical framework. It provides a perturbative formulation of quantum gravity and it incorporates non abelian gauge theories. Our perturbative understanding is well established and leads to the formulation of the celebrated 5 (super)string models (Type IIA/B, $SO(32)$ -Type I, $SO(32)/E_8 \times E_8$ -Heterotic). These theories are (perturbatively) theories of open (type I) and closed (the others) supersymmetric strings in ten dimensional flat space time; such strings vibrate generating (as harmonics) an infinite tower of particles, some of them are massless with appropriate polarization tensors. The effective field theory of such massless fields is a supergravity theory in ten dimension (coupled to Super Yang Mills in the Type I and Heterotic cases).

1.1 D-branes

In these supergravity theories there are black-hole like solutions which are extended in space. These solutions have a definite tension (mass per unit volume) and are charged by the massless p-forms of the corresponding string spectrum / supergravity multiplet. One of the main results of the last decade is the recognition that such supergravity solitons admit a microscopic string theory description: they are Dirichlet branes (D-branes). They can be described as hypersurfaces in space time on which open strings are constrained to end (in this sense $SO(32)$ -type I is a theory of (unoriented) open strings ending on 32 space-time filling D9-branes). However these objects are not just boundary conditions, they are genuine dynamical objects that can move in spacetime; moreover they are physical sources for closed strings. This can be understood in the following way. Imagine to have 2 parallel D-branes and consider an open string connecting the two D-branes, then consider the one loop partition function of this string; graphically this corresponds to a cylinder connecting the two D-branes which, in turn, can be interpreted as an exchange of a closed string between two sources.

This example shows that open and closed strings are deeply intertwined and cannot be studied separately: a theory of open strings generates closed string poles at one loop and, on the other hand, closed strings are sourced by the D-branes on which open strings live on.

The discovery of D-branes has been a key element to understand that the five distinct string theories just mentioned above (plus a still not defined theory, dubbed M-Theory, whose low energy limit is eleven dimensional supergravity) are related to each others by suitable duality transformations. This web of dualities points towards the existence of a single theory to be formulated with a unique and complete set of variables, which reduces to the known superstring theories on particular points of its moduli space. This still hypothetical theory should give a non-perturbative definition

of quantum gravity and, as such, should be background independent: space time itself should arise dynamically as a coherent state from nothing.

1.2 Tachyons

In this scenario the relevance of tachyons is fundamental. From a point particle point of view tachyons are particles that propagate faster than light, violating causality. Equivalently they are relativistic particles with negative $mass^2$. This is however a fake understanding of tachyons and, from a first quantized point of view, a tachyon is just an inconsistency of a theory. On the other hand, in field theory, we know that the concept of mass (of a scalar) arises from the quadratic term of the scalar potential around a stationary point. If the quadratic term is positive then, by quantizing the theory, we get a massive particle but, if the quadratic term is negative, we get a tachyon that simply indicates that we are quantizing the theory on an unstable vacuum, perturbation theory breaks down and some phase transition takes place, driving the theory to a new stable vacuum, with a different perturbative spectrum.

There are many unstable vacua in string theory, signaled by corresponding tachyons on the string spectrum. The simplest example to think about is just the 26 dimensional closed bosonic string in flat space. This theory is not supersymmetric, does not contain fermions (it is in fact not very realistic...) and has a low lying state that is a tachyon. Although this is the simplest tachyon that one encounters in the first study of string theory, it is also the most mysterious one: indeed it signals the instability of the 26 dimensional bosonic spacetime itself and it is not clear at all if some decaying process can bring it to a new stable spacetime (maybe the 10 dimensional supersymmetric spacetimes?).

There is another very simple tachyon, the open string one. A theory of open strings is however a theory of D-branes, as open strings are just excitations of them. If an open string theory contains a tachyon, this can only mean that the corresponding D-brane system is unstable. In particular all bosonic D-branes are not charged and they all contain a tachyon in their spectra. In this sense the 26 dimensional open string tachyon is just the signal of the instability of the space filling D25-brane who does not have any charge protecting it. These examples might look academical as they are in the realm of the bosonic string, there are however other open string tachyons in the supersymmetric string, the bosonic case is just a simpler example of the same kind of phenomenon.

Type-IIA/B theories contain stable D-branes of even/odd dimensions, these branes are stable because they carry RR-charge and source the corresponding RR-massless fields of the closed string sector, they are BPS states and break half of the supersymmetry of the bulk space time. From an open string point of view they don't contain tachyons in their spectra because the NS tachyon has been swept away by the GSO projections needed to keep modular invariance at one loop. Since these branes are charged they possess an orientation given by the corresponding RR-p form which is

a volume form for the brane's worldvolume. A D-brane of opposite orientation is just a D-brane with opposite RR-charge, an anti-D-brane. Now, if a D-brane and an anti-D-brane are placed parallelly at a distance less than the fundamental string length, there is a tachyon corresponding to the lowest state of open strings stretched between the two branes, that arises because such open strings undergo the opposite GSO projection. This tachyon is just the signal that a D-brane/anti-D-brane system is unstable as it does not possess a global RR-charge.

There are also single D-branes which are unstable, these are the branes of wrong dimensionality (odd for Type-IIA, even for Type-IIB), the non-BPS D-branes. The instability is due to the fact that these branes are not charged as there are no RR-p forms in the closed string sector that can couple to them. And there is again the corresponding tachyon in the open string sector coming from the opposite GSO-projection w.r.t. the stable case.

Where these instabilities drive the theory? Is there a stable vacuum to decay to? The answer (at least for the case of open string tachyons) is yes: an unstable system of branes decays to a vacuum where it ceases to exist and its mass is converted in closed string radiation that can propagate in the bulk. This phenomenon is known as Tachyon Condensation.

Why the study of tachyon condensation is important? It is so because the decay of unstable objects is a physical process that interpolates between two different vacua (the unstable one and the stable one). In other words, tachyon condensation is a natural path to explore the (open)-string landscape and, hence, to address in an explicit physical example the study of the elusive concept of background independence.

Needless to say that open string tachyon condensation is just (one of) the starting point(s) for the project of a background-independent formulation of string theory. The mysterious closed string tachyon (who represents the instabilities of space-time itself) still waits for a convincing interpretation. Nevertheless it appears that the physics of open tachyon condensation is still rich enough to get insights into non-perturbative string theory. This is so because of the profound (an not yet fully understood) relation between open and closed strings.

1.3 Open/Closed Duality

One of the latest biggest achievements of string theory is the *AdS/CFT* correspondence which states (in its strongest formulation) that quantum Type-IIB closed string theory on $AdS_5 \times S^5$ with N units of RR 5-form flux is dual to $\mathcal{N} = 4$ $U(N)$ - SYM theory which lives on the projective 4-dimensional boundary of AdS . This correspondence basically states that a (perturbative) quantum theory of gravity on a given background is fully captured by a Yang-Mills theory which is, in turn, the low energy limit of open string theory on N D3-branes in flat space: in other words the open strings dynamics on D-branes in flat space gives us a quantum theory of gravity in a space time which is the result of the back-reaction of the branes on the original

flat geometry. This is not the stating that the full closed string Hilbert space (with all the possible changes in the closed string background) is captured by a particular D-branes configuration, but it means that a complete quantum formulation of open string theory on such D-brane configuration gives a consistent and unitary quantum theory of gravity on a given spacetime. A change in the D-brane system produces a different back reaction, hence a different spacetime. It is maybe too much optimistic to think that all closed strings background can be obtained in this way, but this is certainly an interesting way of thinking at background independence.

1.4 String Field Theories

These examples show that, even if we only know the perturbative expansion around some particular background, there are quite convincing physical reasons to believe that we can understand how string theory backgrounds are dynamically connected. However we have to face the problem that the perturbative formulation of string theory is explicitly non background independent. This in fact is mostly a consequence of the first quantized formulation. History teaches us that the most complete theory of particles has been achieved by passing from first quantization to second quantization, that is from Quantum Mechanics to Quantum Field Theory. It is only in the framework of QFT that one can have control of the vacua of a theory and how such vacua are dynamically connected via nonperturbative effects (tunneling, dynamical symmetry breaking, confinement, etc...). In the theory of particles we see that the right language to describe physics is to promote every particle with a corresponding space-time field and then proceeding with quantization. What about strings? Even in first quantization we see that the quantum fluctuations of a single string give rise to an infinite set of particles: some of them are massless, some of them may be tachyonic and infinite of them are massive. Passing from first quantization to second quantization leads to a QFT with an infinite number of space time fields: the task seems impossible both from a conceptual (infinity means no knowledge in physics) and from a computational (infinite interactions for a given physical process) point of view. However string theory is not just a theory of infinite interacting particles, there is order inside. What marks the difference with respect to particles is the conformal symmetry of the worldsheet theory: this symmetry gives us a consistent and unique interacting scheme (at every order in perturbation theory) starting with non interacting strings. This is like having a rule that gives us (unambiguously) vertices of Feynmann diagrams from the free propagators! In this sense first quantized string theory contains informations about the full non perturbative theory, they are just hidden inside.

A vacuum of string theory is identified once the string propagates in such a way that the corresponding worldsheet theory is conformal, in other words a vacuum of string theory is a two dimensional conformal field theory. What about the string spectrum (the infinite on-shell particles obtained from the vibration of the string)?

They are perturbations of the vacuum, hence they correspond to (infinitesimal) deformations of the underlying conformal field theory. However these are not generic deformations but are such as to preserve conformal symmetry, they are marginal deformations. In this language the string's landscape has an intriguing description: it is the space of two-dimensional field theories. Some points in this space are conformal field theories and correspond to exact string backgrounds (vacua). Around each vacuum there are marginal directions which deform the CFT while maintaining conformal invariance, these infinitesimal deformations are the string excitations around that particular vacuum. Some of these infinitesimal deformations can be exponentiated to a finite one, giving a one parameter family of CFT's/strings vacua. There can be also vacuum points which cannot be connected through (time independent) marginal deformations but that are the result of an RG-flow to some IR fixed point. In this language non-perturbative string theory can be identified with the dynamics of two dimensional field theories. It is evident that such an understanding is equivalent to a second quantized formulation of string theory: a String Field Theory.

In a String Field Theory framework, the basic degrees of freedom are all the possible deformations (string fields) of a given reference conformal field theory one starts with. Such theories admit classical solutions which are in one-to-one correspondence to exact backgrounds of string theory which, in principle, can be completely disconnected from the starting background. They also have, being "field" theories, an off-shell extension of the corresponding first quantized theory: hence they can properly describe non perturbative transitions between different vacua. There are formulations of closed and open string field theories.

While closed string field theory has a complicated non polynomial form that has proven to be resistant to any kind of analytic treatment, Open String Field Theory has a remarkable simple structure which is of Chern-Simons form. The theory is simple enough to do numerical studies on the structure of its vacua. It is fair to say that a complete formulation (where explicit computations can be performed) exists up to now only for the bosonic open string and for the NS sector of the open superstring. In both cases a study of tachyon condensation has been proved possible and a non trivial tachyon potential has been seen to emerge from the level truncated action of (Super) Open String Field Theory.

Very recently the first non trivial solution representing the open string Tachyon vacuum has been derived by Martin Schnabl, [17]. This puts the Open Bosonic String Field Theory in a privileged status as it opens the way for future analytic studies on the non perturbative dynamics of String Theory.

These lectures are meant to be a (very) basic introduction to OSFT, the interested reader is strongly suggested to deepen her/his understanding on the already existing reviews [2, 3, 4, 5, 6, 7]. The list of references we give is by no means complete, the reader should consult the above reviews for a comprehensive set of references.

2 Open String Field Theory: an outline

Open String Field Theory, [1], is a second quantized formulation of the open bosonic string. Its fundamental degrees of freedom are the open string fields, namely all kinds of vertex operators (primary and not primary) that can be inserted at the boundary of a given bulk *CFT*, which represents a (once and for all) fixed closed string background, for example flat space-time.

The explicit action of OSFT is derived starting from the perturbative vacuum representing a given (exactly solvable) Boundary Conformal Field Theory. In most application this *BCFT* is the D25-brane's one, with Neumann boundary conditions on all the (non-interacting) space-time directions.

We will begin being rather formal, concentrating on the abstract properties of the various objects that define the string field theory action. We will then give precise and computable definitions.

2.1 Open Bosonic Strings

To start with, we recall some general feature of the first quantized bosonic string on flat space.

One starts with the σ -model which describes the embedding of the coordinates X^μ from the worldsheet Σ

$$S = -\frac{1}{4\pi\alpha'} \int_{\Sigma} \sqrt{-\gamma} \gamma^{ab} \partial_a X^\mu \partial_b X_\mu \quad (1)$$

Gauge fixing the metric to $\gamma_{ab} = e^\phi \delta_{ab}$ and adding the corresponding ghost/antighost system in order to stay on the gauge slice, leaves us with the combined matter plus ghost CFT

$$S = -\frac{1}{4\pi\alpha'} \int_{\Sigma} d^2z \partial X^\mu \bar{\partial} X_\mu + \frac{1}{\pi} \int_{\Sigma} d^2z (b\bar{\partial}c + \bar{b}\partial\bar{c}) \quad (2)$$

The BRST charge of the above gauge fixing is given by

$$Q = \frac{1}{2\pi i} \oint dz (cT_{matter} + :bc\partial c:) \quad (3)$$

where T_{matter} is the stress tensor of the X^μ CFT (D-free bosons).

It is well known that

$$Q^2 = 0 \quad \Leftrightarrow \quad D = 26 \quad (4)$$

since in this case the total central charge is vanishing and the worldsheet theory is non-anomalous.

The set of physical states is given by vertex operators of ghost number 1 in the cohomology of Q .

$$Q|V_{phys}\rangle = 0, \quad |V_{phys}\rangle \neq Q|W\rangle \quad (5)$$

The previous relation is intended to be applied on insertion of operators on the boundary of the Riemann surface Σ (this is because we are dealing with *open* strings).

Via the state–operator correspondence (which always holds in a radial quantized CFT) we can associate to any operator a corresponding state in the set of boundary deformations of our CFT.

$$|\phi\rangle = \phi(z=0)|0\rangle \quad (6)$$

where $|0\rangle$ is the $SL(2,R)$ invariant vacuum (of ghost number zero).

2.2 Linearized action

We want a space-time action whose variation yields the physical condition coming from the BRST quantization of the worldsheet BCFT.

The job is readily done by the following

$$S_{kin}[\psi] = \langle \psi, Q_{BRST}\psi \rangle \quad (7)$$

In the above formula ψ is a classical string field: a generic vertex operator of ghost number 1; Q_{BRST} is the first quantized BRST operator and the inner product $\langle \cdot, \cdot \rangle$ is the *bpz* inner product, relative to the $BCFT_0$ in consideration (the D25–brane); namely

$$\begin{aligned} \langle \phi, \psi \rangle &= \langle I \circ \phi(0) \psi(0) \rangle_{BCFT_0} \\ I(z) &= -\frac{1}{z} \end{aligned} \quad (8)$$

Note that

$$\langle A \rangle \neq 0 \quad \Rightarrow \quad gh(A) = +3 \quad (9)$$

This condition asks for the classical string field ψ to be at ghost number 1.

By varying the kinetic action we get the linearized equation of motion

$$Q_{BRST}|\psi\rangle = 0 \quad (10)$$

which is the usual on–shell condition for vertex operator of ghost number 1. The action possesses a (reducible) gauge invariance

$$\delta\psi = Q_{BRST}|\Lambda\rangle \quad (11)$$

for a generic string field Λ of ghost number 0, this gauge symmetry is reducible because we can have string fields of any negative ghost number, hence we have to mod out the previous gauge transformation by Q_{BRST} -closed string fields of ghost number zero, and so on. This is achieved using BV quantization (this is very well explained in the review by Thorn).

We have then seen that the non trivial solution of the linearized equation of motion are in one to one correspondence with the usual open string spectrum.

To see how this works practically, let's plug in the action the tachyon vertex operator

$$|\psi\rangle = \int dk t(k) e^{ikx} c_1 |0\rangle \quad (12)$$

Due to the fact that

$$\{Q_{BRST}, b_0\} = L_0 \quad (13)$$

and

$$b_0 |\psi\rangle = 0 \quad (14)$$

We have that

$$\langle\psi|Q_{BRST}|\psi\rangle = \langle\psi|c_0 L_0|\psi\rangle \quad (15)$$

Leading to

$$\langle\psi, Q_{BRST}\psi\rangle = \int dk t(-k)(k^2 - 1)t(k) = \int dx t(x)(\square - 1)t(x) \quad (16)$$

This is nothing but the free space time action for the open string tachyon. In the same way one can get the free actions for the photon and for the other massive modes of the open string.

2.3 The interacting action

The kinetic action describes the small fluctuations of the perturbative vacuum that are identified by the (infinitesimal) boundary marginal deformations of $BCFT_0$. The simplest covariant way to introduce interactions is to add a cubic term to the action

$$S[\psi] = -\frac{1}{g_o^2} \left(\frac{1}{2} \langle\psi, Q_{BRST}\psi\rangle + \frac{1}{3} \langle\psi, \psi * \psi\rangle \right) \quad (17)$$

Note that we have normalized the action with the open string coupling constant g_o .

The cubic term is constructed using the operation $*$ which is an associative non-commutative product in the Hilbert space of string fields.

$$(\psi_1 * \psi_2) * \psi_3 = \psi_1 * (\psi_2 * \psi_3) \quad (18)$$

The Q_{BRST} operator is a derivation of the $*$ -algebra

$$Q_{BRST}(\psi_1 * \psi_2) = (Q_{BRST}\psi_1) * \psi_2 + (-1)^{|\psi_1|}\psi_1 * (Q_{BRST}\psi_2), \quad (19)$$

where $|\psi_1|$ is the grassmannality of the string field, the ghost number in the case of the bosonic string.

We also ask for cicicity

$$\langle A, B * C \rangle = (-1)^{|A|(|B|+|C|)} \langle B, C * A \rangle \quad (20)$$

Using the fact that

$$\langle Q_{BRST}(\dots) \rangle_{BCFT_0} = 0 \quad (21)$$

one can easily prove that the above action is invariant under the following gauge transformation

$$\delta\psi = Q_{BRST}|\Lambda\rangle + [\psi, \Lambda]_* \quad (22)$$

This infinitesimal gauge transformation can be extended to a finite one

$$\psi' = e^\Lambda (Q_{BRST} + \psi) e^{-\Lambda}, \quad (23)$$

where the exponentials are in the $*$ -product sense. The addition of just a cubic coupling to make the action interacting can seem a bit arbitrary and even to much simple. This is however the only consistent choice one can make as it gives a unique and complete covering of the moduli space of Riemann surfaces with boundary, [12]. In other words, any worldsheet of an arbitrary number of external legs and loops can be uniquely recovered by an appropriate Feynmann diagram build with the cubic vertex and the propagator.

The equation of motion are obtained by varying the action with respect to ψ and reads

$$Q_{BRST}|\psi\rangle + |\psi\rangle * |\psi\rangle = 0 \quad (24)$$

Given a solution ψ_0 of the equation of motion one can shift the the string field in the following way

$$\psi = \psi_0 + \phi \quad (25)$$

Then the action can be rewritten as

$$S[\psi] = S[\psi_0] + -\frac{1}{g_o^2} \left(\frac{1}{2} \langle \phi, Q_{\psi_0} \phi \rangle + \frac{1}{3} \langle \phi, \phi * \phi \rangle \right) \quad (26)$$

where the new kinetic operator Q_{ψ_0} is defined

$$Q_{\psi_0} \phi = Q_{BRST} \phi + \{ \psi_0, \phi \}_* \quad (27)$$

The quantity $S[\psi_0]$ is the action evaluated at the classical solution, if this solution is static (it has no kinetic energy), then this quantity corresponds to the static energy of ψ_0 , in particular

$$-\frac{S[\psi_0]}{V^{(26)}} = \tau_{\psi_0} \quad (28)$$

where τ_{ψ_0} is the *tension* of ψ_0 , the space-averaged energy mod space.

3 The $*$ -product

The key element that makes String Field Theory an interacting theory is the promotion of the string field Hilbert space to a non commutative algebra. As already said in the previous chapter this is achieved by introducing a multiplication rule between string fields, the $*$ product. It is time now to explore its definition and its properties. We will first give an heuristic definition based only on the embedding coordinates in the target space $X^\mu(\sigma)$ (the *matter* sector). The matter string field can be understood as a functional of the string embedding coordinates (Schrodinger representation)

$$|\psi\rangle \quad \Rightarrow \quad \psi[X^\mu(\sigma)] = \langle X^\mu(\sigma) | \psi \rangle \quad (29)$$

$$(30)$$

the states $|X^\mu(\sigma)\rangle$ are the open string position eigenstates

$$\hat{X}^\mu(\sigma) |X^\mu(\sigma)\rangle = X^\mu(\sigma) |X^\mu(\sigma)\rangle \quad (31)$$

$$\langle X^\mu(\sigma) | X^\nu(\sigma') \rangle = \eta^{\mu\nu} \delta(\sigma - \sigma') \quad (32)$$

The worldsheet parameter σ spans the whole open string and it lies in the interval $[0, \pi]$. For the definition of the $*$ product it is necessary to split the string into its left and right part, so we define

$$\hat{l}^\mu(\sigma) = \hat{X}^\mu(\sigma) \quad 0 \leq \sigma < \frac{\pi}{2} \quad (33)$$

$$\hat{r}^\mu(\sigma) = \hat{X}^\mu(\pi - \sigma) \quad \frac{\pi}{2} < \sigma \leq \pi \quad (34)$$

The midpoint $\sigma = \frac{\pi}{2}$ cannot be left/right decomposed so we will treat it as a separate coordinate (even if it is a part of a continuum).

$$\hat{x}_m^\mu = \hat{X}^\mu \left(\frac{\pi}{2} \right) \quad (35)$$

given these definitions the string field can be expressed as a functional of the midpoint and left/right degrees of freedom

$$\psi[X^\mu(\sigma)] = \psi[x_m^\mu; l^\mu(\sigma), r^\mu(\sigma)] \quad (36)$$

The bpz inner product can be expressed as a functional integration with respect *all* degrees of freedom

$$\begin{aligned} \langle \psi | \phi \rangle &= \int \mathcal{D}X(\sigma) \langle \psi | X(\sigma) \rangle \langle X(\sigma) | \phi \rangle \\ &= \int \mathcal{D}X(\sigma) \psi[X(\pi - \sigma)] \phi[\sigma] \\ &= \int dx_m \mathcal{D}l(\sigma) \mathcal{D}r(\sigma) \psi[x_m; l(\sigma), r(\sigma)] \phi[x_m, r(\sigma), l(\sigma)] \end{aligned} \quad (37)$$

Note that this operation consists in gluing two strings with opposite left/right orientation. Since all the degrees of freedom are integrated, one is left with just a pure number. This is reminiscent of the trace of the product of two infinite matrices.

The star product between two string fields is another string field defined in the following way

$$(\psi * \phi)[x_m; l(\sigma), r(\sigma)] = \int \mathcal{D}y(\sigma) \psi[x_m; l(\sigma), y(\sigma)] \phi[x_m; y(\sigma), r(\sigma)] \quad (38)$$

This operation consists in identifying the left half of the first string with the right half of the second string, integrating the overlapping degrees of freedom as to reproduce a third string. This is analogous to the multiplication of two infinite matrices.

4 Conformal definition of the cubic vertex

The representation we gave of the star product is very intuitive, but it is not very convenient to do practical computation. Here we want to give a precise definition of the cubic vertex of OSFT based on conformal field theory. Let's consider the most general coupling between 3 string fields

$$\langle \psi, \phi * \chi \rangle = \quad (39)$$

The definitions of the next section imply that we have to compute

$$= \int \mathcal{D}x \mathcal{D}y \mathcal{D}z; \psi[z, y] \phi[y, x] \chi[x, z] \quad (40)$$

This can be interpreted as a correlation function on the 2 dimensional world sheet quantum field theory which is a *conformal* field theory. Using the operator/state correspondence we can define our string fields as generic vertex operators inserted at the origin of the complex upper half plane

$$|A\rangle = A(z=0)|0\rangle \quad (41)$$

Let us consider three unit semidisks in the upper half z_a ($a = 1, 2, 3$) plane. Each one represents the string freely propagating in semicircles from the origin (world-sheet time $\tau = -\infty$) to the unit circle $|z_a| = 1$ ($\tau = 0$), where the interaction is supposed to take place. We map each unit semidisk to a 120° wedge of the complex plane via the following conformal maps:

$$f_a(z_a) = \alpha^{2-a} f(z_a), \quad a = 1, 2, 3 \quad (42)$$

where

$$f(z) = \left(\frac{1+iz}{1-iz} \right)^{\frac{2}{3}} \quad (43)$$

Here $\alpha = e^{\frac{2\pi i}{3}}$ is one of the three third roots of unity. In this way the three semidisks are mapped to nonoverlapping (except at the interaction points $z_a = i$) regions in such a way as to fill up a unit disk centered at the origin. The curvature is zero everywhere except at the center of the disk, which represents the common midpoint of the three strings in interaction.

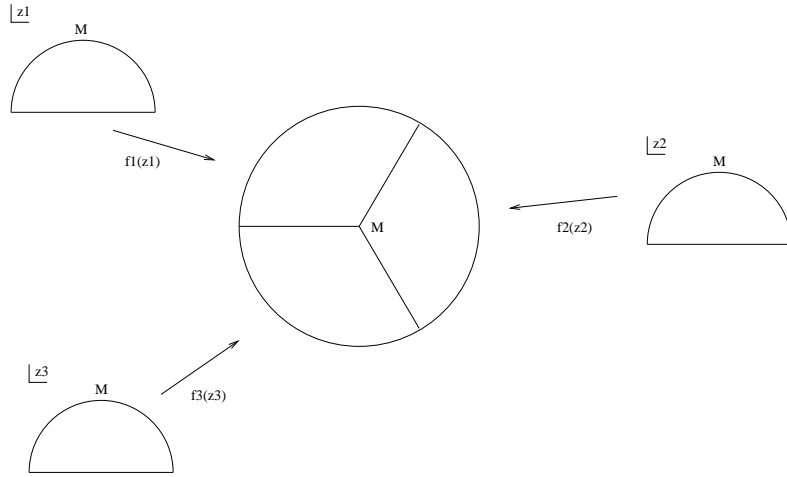


Figure 1: *The conformal maps from the three unit semidisks to the three-wedges unit disk*

The interaction vertex is defined by a correlation function on the disk in the following way

$$\langle \psi, \phi * \chi \rangle = \langle f_1 \circ \psi(0) f_2 \circ \phi(0) f_3 \circ \chi(0) \rangle \quad (44)$$

4.1 The level 0 tachyon potential

Let's see how this work for the zero momentum tachyon, given by

$$|T\rangle = T c(0)|0\rangle \quad (45)$$

We have then to compute

$$\langle T, T * T \rangle = T^3 \langle f_1 \circ c(0) f_2 \circ c(0) f_3 \circ c(0) \rangle \quad (46)$$

Since $c(z)$ is a primary field of weight -1 we have

$$f \circ c(z) = \frac{1}{f'(z)} c(f(z)) \quad (47)$$

In particular, using the explicit definitions and the well known

$$\langle c(z_1) c(z_2) c(z_3) \rangle = (z_1 - z_2)(z_2 - z_3)(z_3 - z_1) \quad (48)$$

we get explicitly

$$\begin{aligned} & \langle f_1 \circ c(0) f_2 \circ c(0) f_3 \circ c(0) \rangle \\ &= \frac{1}{f_1'(0)f_2'(0)f_3'(0)} \langle c(f_1(0)) c(f_2(0)) c(f_3(0)) \rangle = \sqrt{3} \frac{81}{64} \end{aligned} \quad (49)$$

Collecting the result of the kinetic term computation, we can extract the *tachyon potential*, that is (minus) the action for the zero momentum tachyon vertex operator

$$V(T) = -S(T) = \frac{1}{g^2} \left(\frac{1}{2} \langle T, QT \rangle + \frac{1}{3} \langle T, T * T \rangle \right) = M_{D25} (2\pi^2) \left(-\frac{1}{2} T^2 + \frac{1}{3} \sqrt{3} \frac{81}{64} T^3 \right) \quad (50)$$

where we have used the relation between the D25-brane tension and the open string coupling constant ($\alpha' = 1$)

$$M_{D25} = \frac{1}{2\pi^2} \quad (51)$$

The critical points of this potential are at $T = 0$ (*perturbative vacuum*) and at $T = T^* = \frac{64\sqrt{3}}{81}$ (*non-perturbative vacuum*). At the non-perturbative vacuum we get

$$V(T^*) = -0.684 M_{D25} \quad (52)$$

so we already see, although we have truncated our string field to just the tachyon vertex operator (zero level), the energy density of the new vacuum accounts for 68% of the D25 D-brane action!

5 Three strings vertex and matter Neumann coefficients

A very convenient representation for explicit computations with many vertex operators is via the definition of the 3–strings vertex. This object lives on three copies of the string Hilbert space and defines the $*$ product in the following way

$${}_3\langle\psi * \phi| = {}_{123}\langle V_3| |\psi\rangle_1 |\phi\rangle_2 \quad (53)$$

The three strings vertex of Open String Field Theory is given by

$$|V_3\rangle = \int d^{26}p_{(1)} d^{26}p_{(2)} d^{26}p_{(3)} \delta^{26}(p_{(1)} + p_{(2)} + p_{(3)}) \exp(-E) |0, p\rangle_{123} \quad (54)$$

where

$$E = \sum_{a,b=1}^3 \left(\frac{1}{2} \sum_{m,n \geq 1} \eta_{\mu\nu} a_m^{(a)\mu\dagger} V_{mn}^{ab} a_n^{(b)\nu\dagger} + \sum_{n \geq 1} \eta_{\mu\nu} p_{(a)}^\mu V_{0n}^{ab} a_n^{(b)\nu\dagger} + \frac{1}{2} \eta_{\mu\nu} p_{(a)}^\mu V_{00}^{ab} p_{(b)}^\nu \right) \quad (55)$$

Summation over the Lorentz indices $\mu, \nu = 0, \dots, 25$ is understood and η denotes the flat Lorentz metric. The operators $a_m^{(a)\mu}, a_m^{(a)\mu\dagger}$ denote the non–zero modes matter oscillators of the a –th string, which satisfy

$$[a_m^{(a)\mu}, a_n^{(b)\nu\dagger}] = \eta^{\mu\nu} \delta_{mn} \delta^{ab}, \quad m, n \geq 1 \quad (56)$$

$p_{(r)}$ is the momentum of the a –th string and $|0, p\rangle_{123} \equiv |p_{(1)}\rangle \otimes |p_{(2)}\rangle \otimes |p_{(3)}\rangle$ is the tensor product of the Fock vacuum states relative to the three strings. $|p_{(a)}\rangle$ is annihilated by the annihilation operators $a_m^{(a)\mu}$ and it is eigenstate of the momentum operator $\hat{p}_{(a)}^\mu$ with eigenvalue $p_{(a)}^\mu$. The normalization is

$$\langle p_{(a)} | p'_{(b)} \rangle = \delta_{ab} \delta^{26}(p + p') \quad (57)$$

The symbols $V_{nm}^{ab}, V_{0m}^{ab}, V_{00}^{ab}$ are called the Neumann coefficients .

An important ingredient in the following are the bpz transformation properties of the oscillators

$$bpz(a_n^{(a)\mu}) = (-1)^{n+1} a_{-n}^{(a)\mu} \quad (58)$$

Our purpose here is to discuss the definition and the properties of the three strings vertex by exploiting as far as possible the conformal definition given in the previous section. To start with, we consider the string propagator at two generic points of this disk. The Neumann coefficients V_{NM}^{ab} are nothing but the Fourier modes of the propagator with respect to the original coordinates z_a . For the sake of simplicity we will just review the non–zero modes (zero momentum sector).

From the definition of the vertex it is clear that we have

$$V_{nm}^{ab} = \langle V_{123} | a_{-n}^{(a)} a_{-m}^{(b)} | 0 \rangle_{123} \quad (59)$$

On the other hand, from the conformal definition of the vertex

$$\langle V_{123} | a_{-n}^{(a)} a_{-m}^{(b)} | 0 \rangle_{123} = \langle f_a \circ a_n^\dagger f_b \circ a_m^\dagger \rangle \quad (60)$$

This allows us to directly compute the Neumann coefficients, using

$$a_n^\dagger = \frac{1}{\sqrt{n}} \oint dz \frac{1}{z^n} i \partial X(z) \quad (61)$$

Indeed we have

$$V_{mn}^{ab} = \frac{1}{\sqrt{nm}} \oint dz \oint dw \frac{1}{z^n w^m} \langle f_a \circ i \partial X(z) f_b \circ i \partial X(w) \rangle \quad (62)$$

$$= -\frac{1}{\sqrt{nm}} \oint \frac{dz}{2\pi i} \oint \frac{dw}{2\pi i} \frac{1}{z^n} \frac{1}{w^m} f'_a(z) \frac{1}{(f_a(z) - f_b(w))^2} f'_b(w) \quad (63)$$

where we have used the fact that ∂X is a weight 1 primary

$$f \circ \partial X(z) = f'(z) \partial X(f(z)) \quad (64)$$

and the two-point function (propagator)

$$\langle \partial X(z) \partial X(w) \rangle = \frac{1}{(z - w)^2} \quad (65)$$

It is easy to check that

$$\begin{aligned} V_{mn}^{ab} &= V_{nm}^{ba} \\ V_{mn}^{ab} &= (-1)^{n+m} V_{mn}^{ba} \\ V_{mn}^{ab} &= V_{mn}^{a+1, b+1} \end{aligned} \quad (66)$$

In the last equation the upper indices are defined mod 3.

Let us consider the decomposition

$$V_{mn}^{ab} = \frac{1}{3} \left(C_{nm} + \bar{\alpha}^{a-b} U_{nm} + \alpha^{a-b} \bar{U}_{nm} \right) \quad (67)$$

After some algebra one gets

$$\begin{aligned} C_{nm} &= \frac{-1}{\sqrt{nm}} \oint \frac{dz}{2\pi i} \oint \frac{dw}{2\pi i} \frac{1}{z^n} \frac{1}{w^m} \left(\frac{1}{(1+zw)^2} + \frac{1}{(z-w)^2} \right) \\ U_{nm} &= \frac{-1}{3\sqrt{nm}} \oint \frac{dz}{2\pi i} \oint \frac{dw}{2\pi i} \frac{1}{z^n} \frac{1}{w^m} \left(\frac{f^2(w)}{f^2(z)} + 2 \frac{f(z)}{f(w)} \right) \left(\frac{1}{(1+zw)^2} + \frac{1}{(z-w)^2} \right) \\ \bar{U}_{nm} &= \frac{-1}{3\sqrt{nm}} \oint \frac{dz}{2\pi i} \oint \frac{dw}{2\pi i} \frac{1}{z^n} \frac{1}{w^m} \left(\frac{f^2(z)}{f^2(w)} + 2 \frac{f(w)}{f(z)} \right) \left(\frac{1}{(1+zw)^2} + \frac{1}{(z-w)^2} \right) \end{aligned} \quad (68)$$

The integrals can be directly computed in terms of the Taylor coefficients of f . The result is

$$C_{nm} = (-1)^n \delta_{nm} \quad (69)$$

$$U_{nm} = \frac{1}{3\sqrt{nm}} \sum_{l=1}^m l \left[(-1)^n B_{n-l} B_{m-l} + 2b_{n-l} b_{m-l} (-1)^m \right. \\ \left. - (-1)^{n+l} B_{n+l} B_{m-l} - 2b_{n+l} b_{m-l} (-1)^{m+l} \right] \quad (70)$$

$$\bar{U}_{nm} = (-1)^{n+m} U_{nm} \quad (71)$$

where we have set

$$f(z) = \sum_{k=0}^{\infty} b_k z^k \\ f^2(z) = \sum_{k=0}^{\infty} B_k z^k, \quad \text{i.e.} \quad B_k = \sum_{p=0}^k b_p b_{k-p} \quad (72)$$

Eqs.(69, 70, 71) are obtained by expanding the relevant integrands in powers of z, w and correspond to the pole contributions around the origin. We notice that the above integrands have poles also outside the origin, but these poles either are not in the vicinity of the origin of the z and w plane, or, like the poles at $z = w$, simply give vanishing contributions. By changing $z \rightarrow -z$ and $w \rightarrow -w$, it is easy to show that

$$(-1)^n U_{nm} (-1)^m = \bar{U}_{nm}, \quad \text{or} \quad CU = \bar{U}C, \quad C_{nm} = (-1)^n \delta_{nm} \quad (73)$$

In the second part of this equation we have introduced a matrix notation which we will use throughout. One can use this representation for (70, 71) to make computer calculations. For instance it is easy to show that the equations

$$\sum_{k=1}^{\infty} U_{nk} U_{km} = \delta_{nm}, \quad \sum_{k=1}^{\infty} \bar{U}_{nk} \bar{U}_{km} = \delta_{nm} \quad (74)$$

are satisfied to any desired order of approximation. Each identity follows from the other by using (73). Using (74), together with the decomposition (67), it is easy to establish the commutativity relation (written in matrix notation)

$$[CV^{ab}, CV^{a'b'}] = 0 \quad (75)$$

for any a, b, a', b' . This relation is fundamental for the next developments. It is common to define

$$X = CV^{11} \\ X_+ = CV^{12} \\ X_- = CV^{21} \quad (76)$$

Using (74), together with the decomposition (67), it is easy to establish the following linear and non linear relations (written in matrix notation).

$$\begin{aligned}
X + X_+ + X_- &= 1 \\
X^2 + X_+^2 + X_-^2 &= 1 \\
X_+^3 + X_-^3 &= 2X^3 - 3X^2 + 1 \\
X_+X_- &= X^2 - X \\
[X, X_\pm] &= 0 \\
[X_+, X_-] &= 0
\end{aligned} \tag{77}$$

These very important properties encode the associativity of the matter star product.

6 String Field Theory at the Tachyon Vacuum

6.1 Sen's Conjectures

Open String Field Theory is formulated around the D25-brane vacuum, exhibiting an instability due to the presence of the open string tachyon. As reviewed in the introduction, such an instability is understood as the instability of the D25-brane itself. Indeed bosonic D-branes (as well as D-branes anti D-branes pairs and non-BPS D-branes in superstring theory) do not possess any charge which can prevent them from decaying. To see if there is a stable point in the tachyon potential is a task that can be taken over by looking at the space time effective action of string field theory. Explicit numerical computations can be performed if the string spectrum is truncated up to a certain level, so as to have a finite number of spacetime fields. Level truncation is an approximation scheme by which one can recover a more and more precise effective field theory from the exact but somehow formal string field theory action. It consists in expanding the string field up to a certain level (the eigenvalue of the N operator) and in explicitly computing the action using the prescriptions of the previous section to compute $*$ -products and bpz -inner products. A truncated string field takes the form

$$|\Psi\rangle = (\phi(x) + A_\mu(x)a_1^{\mu\dagger} + \dots)c_1|0\rangle \tag{78}$$

Plugging this expression in the action one ends up with a local action for the component fields up to a certain level

$$\mathcal{S}(\Psi) = \int d^{26}x F(\varphi_i, \partial\varphi_i, \dots) \tag{79}$$

This action is a purely spacetime action and one can extract from it an effective tachyon potential.

Here we do not attempt at all to give a review of the level truncation computations from which the tachyon potential has been obtained, we just quote that a strong

evidence that a local minimum exists has been achieved (see [2] for a pedagogical review of the level truncation technique). By truncating the action at a finite level one ends up with an effective tachyon potential, that have the qualitative form showed in figure

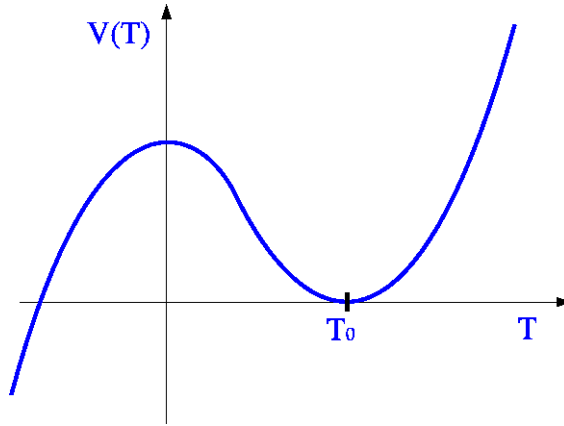


Figure 2: *The tachyon potential of open string field theory: the local maximum represents the unstable D25-brane, while the local minimum is the tachyon vacuum*

Lot of computations has been done to assure that the following statements about the tachyon vacuum are true

- The energy difference between the perturbative vacuum and the tachyon vacuum exactly matches with the D25-brane energy, hence it represents a configuration with no D-branes at all,
- The cohomology around the tachyon vacuum is trivial (at least at ghost number one), indicating that there are no physical perturbative open string states around it
- Lower dimensional D-branes can be obtained as tachyonic lumps in which, along the transverse directions, the tachyon reaches its minimum in the potential at $\pm\infty$ and it is vanishing at the origin, the energy of such lump solutions matches with the lower dimensional D-branes energy.

Such statements are known as Sen's conjectures and, thanks to the great amount of evidence reached, they are universally accepted as fundamental properties.

6.2 OSFT with no open strings

The second Sen's conjecture states that at the tachyon vacuum the theory we get is rather strange. In particular it states that there are no physical excitation. This is a

spectacular phenomenon that has no counterpart in usual local field theories. This is well understandable from a physical point of view. We are dealing with a theory of open strings, and open strings states are what we call the physical excitations. Now, if we are sitting in a vacuum with no D-branes at all we cannot expect to have a physical open string spectrum, as open strings cannot attach anywhere. How can the tachyon vacuum BRST charge be trivial when the one we are starting with is not? The answer can be found in non trivial nature of the classical solution describing the tachyon vacuum. We remind that given a classical solution of OSFT, the induced BRST charge / kinetic term is given by

$$Q_{\psi_0}\phi = Q_{old}\phi + \psi_0 * \phi + \phi * \psi_0 \quad (80)$$

Let's now introduce the identity string field, $|I\rangle$. By definition it is the identity of the star algebra.

$$|I\rangle * |\phi\rangle = |\phi\rangle * |I\rangle = |\phi\rangle \quad (81)$$

One can now show that if there exists a string field A of ghost number -1 such that

$$I = Q_{\psi_0}A, \quad (82)$$

then the cohomology of Q_{ψ_0} is empty at any ghost number. The proof of this very important statement is embarrassingly simple

$$0 = Q_{\psi_0}\phi = A*Q_{\psi_0}\phi = -Q_{\psi_0}(A*\phi) + (Q_{\psi_0}A)*\phi = -Q_{\psi_0}(A*\phi) + I*\phi = -Q_{\psi_0}(A*\phi) + \phi \quad (83)$$

that is

$$Q_{\psi_0}\phi = 0 \quad \Leftrightarrow \quad \phi = Q_{\psi_0}(A * \phi) \quad (84)$$

This string field A has been recently found by Ellwood and Schnabl, [16], and is based on the first non trivial classical solution representing the tachyon vacuum given by Schnabl [17].

6.3 Vacuum String Field Theory

The remarkable properties of the tachyon vacuum suggest that Open String Field Theory should take its simplest form around it. As we have seen in section 2, when we expand OSFT around a classical solution the action is reproduced up to a shift in the kinetic operator, (27). So the only thing we need to write down OSFT at the tachyon vacuum is the new kinetic operator which, in turn, is known if the classical solution representing the tachyon vacuum is known. Alternatively one can use Sen's conjectures to guess the form of the kinetic operator. In [14] a conjecture was put forward under the name of Vacuum String Field Theory. In this model the BRST operator is taken to be pure ghost: this is a particular implementation of the

universality of the tachyon vacuum. In particular the proposed kinetic operator takes the form of a c -midpoint insertion, [13]

$$\mathcal{Q} = \frac{1}{2i} (c(i) - c(-i)) \quad (85)$$

We recall that this operator has trivial cohomology due to the relation

$$\{\mathcal{Q}, b_0\} = 1 \quad \Rightarrow \quad \mathcal{Q}\psi = 0 \rightarrow \psi = \mathcal{Q}(b_0\psi) \quad (86)$$

Since both the star product and the kinetic operator are matter–ghost factorized, it is natural to search for solutions of the equation of motion which are matter/ghost factorized too. In particular, starting from the VSFT equation of motion

$$\mathcal{Q}\psi + \psi * \psi = 0 \quad (87)$$

and making the factorization ansatz

$$\psi = \psi_m \otimes \psi_{gh} \quad (88)$$

one ends up with the following equations, in the ghost and matter sector

$$\mathcal{Q}\psi_{gh} + \psi_{gh} *_{gh} \psi_{gh} = 0 \quad (89)$$

$$\psi_m *_{m} \psi_m = \psi_m \quad (90)$$

The equation in the matter sector defines idempotents (projectors) of the matter star algebra. One very interesting topic is the classification of projectors of the star algebra, as it corresponds to a classification of D–branes. From this analysis it emerges that a single D–brane can be described by a rank 1 projector and a set of N D–branes by a rank N projector (that is the sum of N orthogonal rank 1 projectors).

Rank 1 projectors can be easily defined starting from split left/right open string functionals

$$\psi[X(\sigma)] = \psi[l(\sigma), r(\sigma)] = \psi_L[l(\sigma)]\psi_R[r(\sigma)] \quad (91)$$

When we compute the $*$ –square we get

$$(\psi_L[l(\sigma)]\psi_R[r(\sigma)]) * (\psi_L[l(\sigma)]\psi_R[r(\sigma)]) = K \psi_L[l(\sigma)]\psi_R[r(\sigma)] \quad (92)$$

where (formally)

$$K = \int \mathcal{D}y(\sigma) \psi_R(\sigma)\psi_L(\pi - \sigma), \quad \sigma \in \left[\frac{\pi}{2}, \pi\right] \quad (93)$$

So, if ψ_L and ψ_R are chosen in such a way that $K \neq 0$ a rank 1 projector is simply given by $\frac{1}{K}\psi$. One very simple choice is to choose twist invariant states ($L \leftrightarrow R$), that is

$$\psi_L[l(\sigma)] = \psi_R[r(\pi - \sigma)] \quad (94)$$

with the normalization condition on ψ_R

$$\int \mathcal{D}y(\sigma) \psi_R[y(\sigma)] \psi_R[y(\sigma)] = 1 \quad (95)$$

It is clear that such a state projects into a 1 dimensional subspace of the half string Hilbert space, spanned by all possible functions $r(\sigma)$. It is also clear how to construct other projectors that are orthogonal wrt it, just choose another half string functional $\phi_R[r(\sigma)]$ such that

$$\int \mathcal{D}y(\sigma) \psi_R[y(\sigma)] \phi_R[y(\sigma)] = 0 \quad (96)$$

This means that for every function $f(\sigma)$ I can construct the "characteristic" half string functional ψ_f

$$\psi_f[y(\sigma)] = \delta[f(\sigma) - y(\sigma)] \quad (97)$$

Each characteristic functional defines a projector and characteristic functionals relative to different functions are of course orthogonal.

This quite abstract construction of star algebra projectors can be made very precise using the conformal or the operatorial definition of the star product, see the reviews in the references.

As a last remark we would like to point out that star algebra projector can be used not just as solutions of VSFT (which is in some sense too much singular and needs some not yet understood regularization) but they are also relevant for solving the original OSFT equation of motion, [18].

6.4 Closed Strings in OSFT

So far we have been dealing with open strings and D-branes. We know that this is not the end of the story as unitarity forces us to include a closed string sector. One of the biggest results in the eighties in OSFT was that one loop non planar SFT diagrams contained on shell closed string poles. This lead to the believing that closed strings can be a derived concept, the fundamental degrees of freedom being the open string fields. Are closed strings elementary excitations? or they are maybe automatically generated when one is dealing with the full Quantum Open String Field Theory. The reader might recognize that the answer to this question can provide a microscopic derivation of the gauge/gravity correspondence. At the moment there are not satisfactory proof of this, nonetheless some progresses have been made in this direction.

OSFT is a gauge theory and, as in any gauge theory, a prominent role is de to gauge invariant operators.

It is simple to show that to any *on shell* closed string state we can associate a gauge invariant operator by inserting the closed string vertex operator at the *midpoint*

of the identity string field. In particular let's consider the following second quantized operator/ functional

$$\mathcal{O}_V[\psi] = \langle I|V(\pi/2)|\psi\rangle \quad (98)$$

It is easy to show that this operator is gauge invariant if and only if

$$\{Q, V(\pi/2)\} = \{\bar{Q}, V(\pi/2)\} = 0 \quad (99)$$

that is $V(\sigma)$ is an on-shell closed string vertex operator. One can show that the Feynmann rules derived by adding such gauge invariant operators to the OSFT action gives a complete covering of Riemann surfaces with open plus closed string punctures and at least one boundary. This physically means that with the use of just open string variables one can compute the scattering of closed strings off a brane using open string propagators and cubic open string vertices. As in AdS/CFT closed strings are viewed as non dynamical sources which are needed by gauge invariance.

The question is now: can we get *purely* closed string amplitudes from OSFT? The naive answer to this question is no, we cannot: any OSFT diagram will always contain at least one boundary, where there are the boundary conditions of the D-brane we are working on. But now we know that at the tachyon vacuum there are no D-branes, so what happens to the boundary? One can show, [13], that due to the suppression of open string modes at the tachyon vacuum, the integral over the Schwinger parameters defining the lengths of the propagators are localized to zero length. This in turn implies that any boundary collapses to zero size as well, becoming a closed string puncture. This very heuristic picture can be made more precise: The scattering of n on-shell closed string states off the Tachyon Vacuum is identical to the scattering of $n+1$ closed strings with no D-branes, the extra closed string state being given by the open string boundary which shrinks to zero size. So we are maybe near to a non-perturbative and *computable* definition of String Theory (at least for the bosonic case...)

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