ISAPP school 21

DARK MATTER FROM STANDARD MODEL AND BEYOND, A SELECTION OF PRODUCTION MECHANISMS.

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PART II

IV the FREEZE-OUT MECHANISM. DS & MO.

driven by (co-) annihilations.

Let us consider first, the "textbook" case of a DM particle that is thermal = kinetic equilibrium with the heat bath at early time*.

In the latter case, since early times, Y_χ will follow Y_χ^{eq} up until when chemical decoupling will happen. At that point, if no other number changing processes light on, we expect Y_χ freeze. = FREEZE-OUT.

In order to evaluate the chemical decoupling temperature, T_{CD}, one uses $\boxed{\Gamma_{ann} = H(T_{CD})}$

If T_{CD} happens in radiation dominated era (ie $T_{CD} > T_{eq}$ and no early matter dominated epoch)

$$H = \frac{T^2}{M_0(T)} \quad \text{where} \quad M_0(T) \sim \frac{M_P}{g_*^{1/2}}$$

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* Let us emphasize that the heat bath is usually the SM bath, however, the dark sector could very well be thermally decoupled with $T' \neq T$ (DS, VS), see e.g. 2105.01263

In these lectures, we refer to T as the SM heat bath temperature.

We have multiple examples of particles in kinetic and chemical equ. at early times that decouple at later times.

The most obvious one is the SM neutrino, ν_L.

$\longrightarrow$ we know that they have a non zero hypercharge and isospin combined to give $Q_\nu = 0$

$\longrightarrow$ ν_L interacts with weak interactions with the SM heat bath at early times

Let's assume for a minute that we do not know about laboratory and cosmology constraints and let's try to evaluate for which mass m_ν the SM neutrinos could account for all the DM.

In the next sections, we will go step by step. Let us however flash the final result for the ν_L abundance shown on p. 8.

IV.1 Hot relics ≡ Freeze-out while relativistic

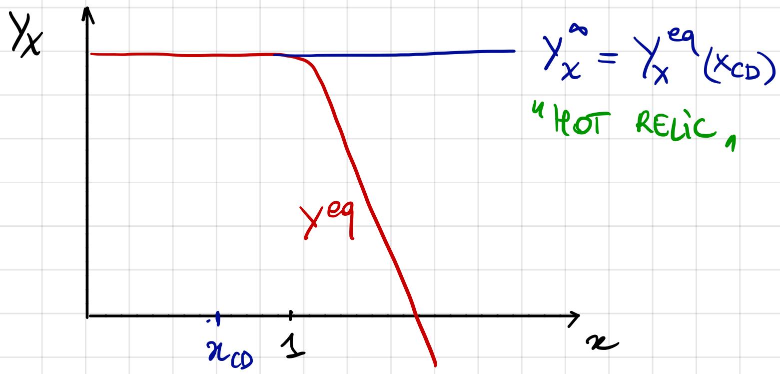
In the case where our χ particle would be sufficiently coupled to the heat bath at early time ($T \gg m_\chi$) <u>and</u> if the particle interactions are such that $T_{CD} > m_\chi$ the particle will decouple while still being relativistic

$$\Rightarrow \quad Y_\chi^\infty = Y_\chi^{eq}(x_{CD}) = \frac{n_\chi(T_{CD})}{s(T_{CD})}$$

$$= 0{,}278 \frac{g_\chi^n}{h_{eff}(T_{CD})}$$

$$\Rightarrow \quad \boxed{\Omega_\chi^0 h^2 = \frac{Y_\chi^\infty}{\rho_c^0} s_0 m_\chi = 0{,}12 \left(\frac{g_\chi^n \, m_\chi}{6\,\mathrm{eV}}\right) \frac{h_{eff}^0}{h_{eff}(T_{CD})}}$$

FO HOT RELIC



This type of hot relics is exactly the case of SM neutrinos.

In the case of a 2 dof fermion DM *

i.e. $g_X^n = \frac{3}{4}\, 2$

in the form of DM interacting through weak interactions:

i.e. $\Gamma_{ann} \sim \langle\sigma v\rangle n_X \sim G_F^2 T^5$ for $m_X \ll T_{CD}$,

$\Rightarrow \Gamma_{ann}(T_{CD}) = H(T_{CD}) \Rightarrow T_{CD} \sim O(\text{MeV})$

$\Rightarrow h_{eff}(T_{CD}) \simeq g_\gamma^s + g_e^s + \underset{N_\nu}{3}\, g_\nu^s = 10.75$.

$$\Rightarrow \boxed{\Omega_X h^2 = \left(\frac{g_X}{2}\right) \frac{m_X}{92\,\text{eV}} \qquad \text{HOT RELIC EWly interacting fermion}}$$

A well known bound on thermal DM is obtained imposing that the HOT RELIC should satisfy $\Omega_X h^2 < 1 \Rightarrow m_X < 92\,\text{eV}$.
= COSWIK AND Mc Clelland bound

For $\Omega_X h^2 < 0{,}12 \Rightarrow m_X = \sum_\nu m_\nu \lesssim 10\,\text{eV}$.

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* SM neutrinos are either Majorana, in which case $g_\nu^M = 2$ or Dirac, but the $\nu_R = \nu_{sterile}$ does not couple to Z boson, so that again $g_\nu^D = 2$

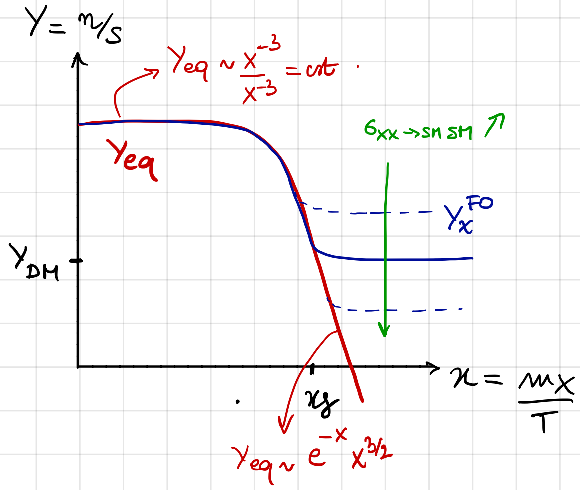
IV.2. Cold Relics = F.O. while Non relativistic

This is the general case of the so-called WIMP, weakly interacting massive particles.

Let us emphasize the "weakly interacting" is not = "$SU(2)_L$ interacting" but more generally "interacting with $g \sim \mathcal{O}(g_{SU(2)_L})$"

For cold relics, it is assumed that DM decouples when N.R. ie while $Y_\chi \sim e^{-x} x^{3/2}$

In the latter case the DM number density evolution goes as follows:

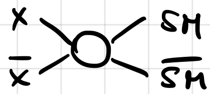


IV 2.1. Vanilla WIMP : approximate result

Here, we just rapidly evaluate the DM relic abundance in the instantaneous freeze-out approximation.
A more precise evolution can be found in. Gondolo - Gelmini '91.

Consider the following process $\chi\bar{\chi} \leftrightarrow SM\,\overline{SM}$ for keeping χ in equilibrium :

$$\Gamma_A = n_\chi \langle \sigma_A . v \rangle$$



$\langle \cdots \rangle \equiv$ thermal average.

- We first estimate the time (or temperature) at which the freeze-out occurs :

$$\Gamma_A \simeq H(T_{dec})$$

$n_\chi = n_\chi^{N.R.}$

$$\Leftrightarrow \langle \sigma_A . v \rangle . \; g_\chi \left(\frac{m_\chi T_{dec}}{2\pi} \right)^{3/2} \exp\left(-\frac{m_\chi}{T_{dec}} \right) \simeq 1.66 \sqrt{g_*} \, \frac{T_{dec}^2}{M_p}$$

$$\Leftrightarrow \langle \sigma_A . v \rangle \, g_\chi \left(\frac{m_\chi^2}{x_f} \right)^{3/2} \exp(-x_f) \simeq \frac{m_\chi^2}{x_f^2 M_p}.$$

$x_f = \frac{m_\chi}{T_{CD}}.$

$$\Leftrightarrow \langle \sigma_A v \rangle \, m_\chi . M_p \sqrt{x_f} \simeq \exp x_f .$$

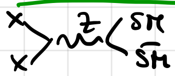
$$\Rightarrow \quad x_f \simeq \frac{1}{2} \ln x_f + \ln(\langle \sigma_A v \rangle m_\chi M_p)$$

Considering $m_\chi \gg m_{SM}$ you can have

$$\langle \sigma_A v \rangle \sim \frac{1}{m_\chi^2} = cst$$

Also considering $m_\chi \ll M_p$ you can approximate :

$$\boxed{\begin{aligned} x_f &\cong \log\left(\langle \sigma_A v \rangle m_\chi \cdot M_p\right) \\ \text{and } \; T_{CD} &\doteq \frac{m_\chi}{x_f} < m_\chi . \end{aligned}}$$

Considering 

: $m_\chi = \begin{cases} 10\,\text{GeV} \\ 260 \end{cases} \rightarrow x_f = \begin{cases} 20 \\ 25 \end{cases}$

- On the other hand

$$\Gamma_A = H(T_{CD}) \Rightarrow n(T_{CD}) = \frac{1}{\langle \sigma_A v \rangle} \frac{T_{CD}^2}{M_p}$$

$$\& \quad n(T_0) = n(T_{CD}) \frac{a_{CD}^3}{a_0^3} \sim n(T_{CD}) \frac{T_{CD}^{-3}}{T_0^{-3}}$$

$$\Rightarrow \quad \Omega_\chi h^2 \sim n_\chi(T_0) m_\chi \propto \frac{m_\chi}{\langle \sigma_A v \rangle M_p T_{CD}},$$

$$\text{i.e. } \Omega_\chi h^2 \propto \frac{x_f}{\langle \sigma_A v \rangle M_p}.$$

IV 2.2. Vanilla WIMP : detailed calculations

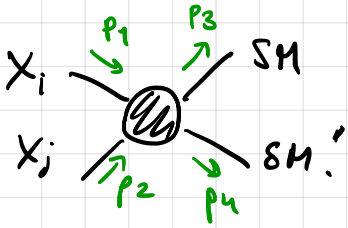
Going back to the Boltzmann equations one can recover the exact result for $\Omega_\chi h^2$.

Let me work as in the previous set-up, where DM interacts with SM dof without caring much about potential dark sector or visible sector mediators. I refer to this case as Vanilla WIMP and calculation details can be found in Gondolo Gelmini '91

Here, we are going to assume that elastic χ SM $\leftrightarrow$ χSM and inelastic (annihilations) $\chi\chi \leftrightarrow$ SM SM are happening fast enough to ensure that DM is in kinetic equ (we can use (FD, BE, MB distributions) and chemical equ in the early universe. Assuming that χSM $\leftrightarrow$ χSM decouple after $\chi\chi \leftrightarrow$ SM SM, we can focus on the Boltzmann equ involving $\chi\chi \leftrightarrow$ SM SM only. We are also going to neglect spin statistic factors $(1 \pm f_{fin})$.

We can integrate over 3-momentum of one DM particle, say particle of momentum p_1, both

df/dt and $\frac{1}{E_1} C[f]$



and $f_1 = f_X(p_1)$

$f_3 = f_{SM}(p_3)$

- $\int \frac{d^3p_1}{(2\pi)^3} \frac{d f_X(p)}{dt} = \frac{dn_X}{dt} + 3H n_X$

- we also assume that the SM are in chemical and kinetic eq with the SM plasma. In that framework, energy conservation, $E_i + E_j = E_{SM} + E_{SM'}$, implies:

$$f_1^{eq} f_2^{eq} = f_3^{eq} f_4^{eq}$$

So that:

$$\int \frac{d^3p_1}{(2\pi)^3} \frac{1}{E_1} C[f_X]$$

Boltzmann eqs of p26.

$$\overset{\downarrow}{=} \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \sigma_{12} v_{12} \left(f_1^{eq} f_2^{eq} - f_1 f_2\right).$$

In order to go 1 step further, we are going to assume that X_1 stays in kinetic equilibrium even for $T \gtrsim T_{CD}$.
We use:

$$\boxed{\frac{f_i(p_i,t)}{f_i^{eq}(p_i,t)} = \exp\left(\frac{-\mu(t)}{T}\right) = \frac{n_i(t)}{n_i^{eq}(t)}}$$

As a result we have

$$\int d^3p_1 \, \frac{1}{E_1} C[f] = \langle \sigma_{12} v_{12} \rangle \left(n_1^{eq} n_2^{eq} - n_1 \, n_2 \right)$$

where the thermally averaged cross-section is:

$$\boxed{\begin{aligned} \langle \sigma_{12} v_{12} \rangle &= \frac{1}{n_1^{eq} n_2^{eq}} \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} \, \sigma_{12} \, v_{12} \, f_1^{eq} \, f_2^{eq} \\ v_{ij} &= \frac{\sqrt{(\overbrace{p_i \cdot p_j}^{\text{4-momentum}})^2 - m_i^2 m_j^2}}{E_i \, E_j} \end{aligned}}$$

and v_{ij} is referred to as the Møller velocity that we have defined in a Lorentz invariant way

At the end of the day, the integrated Boltzmann equ. reads:

$$\boxed{\frac{dn_1}{dt} + 3 H n_1 = \langle \sigma_{12} v_{12} \rangle \left(n_1^{eq} n_2^{eq} - n_1 n_2 \right).}$$

This equation can easily be rewritten in terms of the dimensionless variables:

$$\boxed{\begin{aligned} \frac{dY_1}{dx} &= \frac{\langle \sigma_{12} v_{12} \rangle}{\bar{H} x} \left(Y_1 Y_2 - Y_1^{eq} Y_2^{eq} \right) \\ \text{where} \quad \bar{H} &= H \left(1 + \frac{1}{3} \frac{d\ln h_{eff}}{d\ln T} \right)^{-1} \end{aligned}}$$

$\bar{H}$ comes from the fact that we assume that entropy is conserved

$$\frac{d(a^3 s)}{dt} = 0 \quad \text{and} \quad s \propto h_{eff} T^3$$

$$\Rightarrow \boxed{\frac{d\ln T}{d\ln t} = -\bar{H}}$$

Keeping in mind that for DM self annihilation $m_i = m_j$, we have

$$\boxed{\frac{dn}{dt} + 3Hn = \langle \sigma_A v \rangle \left(n_{eq}^2 - n^2 \right)}$$

VANILLA WIMP.

this is valid for both $\chi = \bar{\chi}$ and $\chi \neq \bar{\chi}$

- One can also define the relative velocity v_{rel} as:

$$v_{rel} = \frac{\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}}{s} \neq v_{mol}$$

In the center of mass frame $v_{rel} = \frac{4\,|\vec{p}_{i\,cm}|}{\sqrt{s}}$

- Notice though that for $m_i = m_j \Rightarrow s = (2E_i)^2$
$\Rightarrow \quad v_{mol} = v_{rel} = v$

In the latter framework, in the N.R. lim

$$\sigma_A v = a + b v^2 + \dots$$

a = s-wave term, b = p-wave term, ...

$$\Rightarrow \langle \sigma_A v \rangle \underset{N.R.}{=} \sqrt{\frac{x^3}{4\pi}} \int_0^\infty dv\, v^2\, e^{-\frac{x v^2}{4}}\, \sigma_A v$$

and $$\boxed{\langle \sigma_A v \rangle = a + \frac{6b}{x} + \dots}$$

It can be shown that for $x > x_{CD} = x_f$

$$\boxed{\Omega_\chi h^2 = 0{,}12 \left(\frac{2.2\, 10^{-26}\ \mathrm{cm^3/s}}{a + 3b/x_f + \dots} \right) \sqrt{\frac{80}{g_*}}\ \frac{x_f}{23}}$$

and we have assumed $g_* = cst$ and $h_{eff} = cst$

- Let us take the example of DM $\equiv \nu_2$. For $m_\nu >$ MeV and $m_\nu < m_Z \Rightarrow \sigma_A v \propto G_F^2 m_\nu^2$

$$\Rightarrow \Omega_\nu h^2 \sim \frac{1}{m_\nu^2}$$

and going through the details of the cross-section, one would get the right DM abundance for $m_\nu = 6$ GeV for a COLD relic

- Unitarity limits the annihilation cross-section reads as.

$$\boxed{\sigma_A v < \frac{4\pi\,(2J+1)}{m_\chi^2\, v}}$$

Griest & Kamionkowski '90. [GK]

for s-wave annihilation taking

$$v|_{FO} \sim \sqrt{\langle v^2\rangle} = \sqrt{\frac{6}{x_f}}$$ with $x_f = 25$

and $\sigma_A v = 2.2\, 10^{-26}\, cm^3/s$

$\Rightarrow$ $m_\chi \sim 100$ TeV. saturates the unitary bound.

One expect thus at high mass

$$\Omega_\nu h^2 \sim \frac{1}{\sigma_A v}\Big|_{GK} \sim m_\nu^2$$

see also 2105.01263 for a generalisation with a HS at T' or VS at T.

$\rightarrow$ higher mass unitary bound if $T' < T$

IV.2.3 Co-annihilations. (Griest & Seckel '91)

from WIMP to FIMP.

"weak" interactions

"feeble" interactors

Let us now assume that the DM χ is not the only Z_2 odd particle. Let us introduce $\chi_i = 1 \dots N$; $m_i > m_j$ for $i > j$ and $\chi_1 = DM = \chi$.

If these dark sector (Z_2 odd) particles are close in mass with χ, they will affect the final DM abundance.

In order to determine the DM relic abundance, one should a priori take into account all

- self- / co- annihilation processes with eg.

$$\sigma_{ij} = \sigma(\chi_i \chi_j \to SM\, SM')$$

- conversion processes and elastic scatterings, decays and inverse decays, with eg.

$$\begin{cases} \sigma_{i\to j} = \sigma(\chi_i\, SM \to \chi_j\, SM') \\ \Gamma_{i\to j} = \Gamma(\chi_i \to \chi_j\, SM) \end{cases}$$

- The generic form of the N. Boltzmann equations would be:

$$\frac{dn_i}{dt} + 3H n_i = - \sum_{j=1} \langle \sigma_{ij} v_{ij} \rangle \left(n_i n_j - n_i^{eq} n_j^{eq} \right)$$
$$- \sum_{j \neq i, SM} n_{SM}^{eq} \Big(\langle \sigma_{i \leftrightarrow j} v_{iSM} \rangle (n_i - n_i^{eq}) - \langle \sigma_{j \to i} v_{jSM} \rangle (n_j - n_j^{eq}) \Big)$$
$$- \sum_{j \neq i} \Big(\langle \Gamma_{i \to j} \rangle (n_i - n_j^{eq}) - \langle \Gamma_{j \to i} \rangle (n_j - n_j^{eq}) \Big)$$

- Side note

Notice that we can rewrite the above eqs in a more compact form using the reaction rates for $2 \leftrightarrow 2$ and $1 \leftrightarrow$ processes as:

$$\gamma_{ij \to kl} = \iint d\phi_i d\phi_j \, f_i^{eq} f_j^{eq} \int d\phi_k d\phi_l \times (2\pi)^4 \delta^4 (p_i + p_j - p_k - p_l) |M_{ij \to kl}|^2$$
$$= n_i^{eq} n_j^{eq} \langle \sigma_{ij} v_{ij} \rangle$$

$$\gamma_{k \to ij} = \int d\phi_k \, f_k^{eq} \times (2\pi)^4 \delta^4 (p_i + p_j - p_k) |M_{k \to ij}|^2$$
$$= n_k^{eq} \langle \Gamma_{k \to ij} \rangle$$

where $d\phi_i = \dfrac{d^3 p_i}{(2\pi)^3 2E_i}$; i, j, k, l denote both SM and DS particles.

$$\langle \Gamma_{k \to ij} \rangle = \Gamma_{k \to ij} \underbrace{K_1(x) / K_2(x)}_{\langle 1/\gamma \rangle}$$ = thermally averaged decay rate.

Assuming no ~~CP~~, we get for dimensionless variables.

$$\boxed{\begin{aligned} H x s \frac{dY_i}{dx} = & -\sum_{jk} \gamma_{ij\to kl} \left(\frac{Y_i Y_j}{Y_i^{eq} Y_j^{eq}} - \frac{Y_k Y_l}{Y_k^{eq} Y_l^{eq}} \right) \\ & - \sum_{jk} \gamma_{k\to ij} \left(\frac{Y_i Y_j}{Y_i^{eq} Y_j^{eq}} - \frac{Y_k}{Y_k^{eq}} \right). \end{aligned}}$$

• (1) Assuming that short after DM FO, all $\{X_i\}_{i\neq 1}$ decay to DM $= X_1$, one can write for

$$n = \sum_{i=1}^{N} n_i$$

$$\frac{dn}{dt} + 3Hn = \sum_{ij=1}^{N} \langle \sigma_{ij} v_{ij} \rangle \left(n_i n_j - n_i^{eq} n_j^{eq} \right).$$

(2) Here we assume that all the relevant particles are in thermal and chemical equ in the early universe. In particular, assuming $x_i \leftrightarrow x_j$ conversion processes happen fast enough ($\Gamma_{x_i \leftrightarrow x_j} > H(x_f)$) so as to consider them in chemical equ.

ie

$$\mu_i \simeq \mu_j \Rightarrow \frac{n_i}{n_i^{eq}} = e^{-\mu_i/T} = e^{-\mu_j/T} = \frac{n_j}{n_j^{eq}}$$

and

$$\frac{n_i}{n_i^{eq}} = \frac{n}{n^{eq}} \quad \text{with } n_{eq} \simeq \sum_i g_i \int \frac{d^3 p_i}{(2\pi)^3} e^{-E_i/T}$$

This allows to simplify even more the Boltzmann eq., which reduces to:

$$\boxed{\begin{aligned} &\dot{n} + 3Hn = -\langle \sigma_{eff} v \rangle \left(n^2 - n_{eq}^2 \right) \\ &\text{with } \langle \sigma_{eff} v \rangle = \sum_{ij} \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{eq} n_j^{eq}}{n^{eq}\, n^{eq}} \end{aligned}}$$

In particular, when considering the DM χ_1 and a Z_2 odd partner χ_2, we have 3 type of possibilities:

$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma v_{eff} \rangle} \propto \begin{cases} \dfrac{1}{\langle \sigma_{11} v \rangle} & \sigma_{11} \gg \sigma_{12}, \sigma_{22} \quad \equiv \text{DM annihil. F.O.} \\ \dfrac{1}{\langle \sigma_{12} v \rangle \exp(-\Delta x)} & \sigma_{12} \gg \sigma_{11}, \sigma_{22} \quad \equiv \text{co-annihilation F.O.} \\ \dfrac{1}{\langle \sigma_{22} v \rangle \exp(-2\Delta x)} & \sigma_{22} \gg \sigma_{11}, \sigma_{12} \quad \equiv \text{mediator ann. F.O.} \end{cases}$$

The relative mass difference $\Delta = \dfrac{m_1 - m_2}{m_1}$ should not be too small for co-annihilations to play a role.

NB Here we see in particular that in the case in which χ_1 coupling to SM and χ_j would be smaller than the ones of χ_j to e.g. SM but still large enough to keep thermal or chemical equ., one could very well have the DM relic abundance fully driven by χ_j annihilation if $\Delta_j = \frac{m_j - m_1}{m_1}$ is small enough.

In this extreme case, that I refer to as "mediator annihilation FO", one has for one extra DS particle χ_2:

$$\langle \sigma_{eff} v \rangle\Big|_{\substack{med \\ ann}} = \langle \sigma_{22} v_{22} \rangle \frac{g_2^2}{g_{eff}} (1+\Delta_i)^3 e^{-2x\Delta_2}$$

$$g_{eff} = \sum_i g_i (1+\Delta_i)^{3/2} \exp(-x\Delta_i)$$

i.e. the crossection setting the relic abundance is "independent" of $\chi_1 \leftrightarrow$ SM interactions

IV 3. DM-, co-, mediator-annihilation FO, one illustrative case:

Let us illustrate the transition in DM–SM coupling strength from "vanilla WIMP" to the case of mediator annihil FO. in the case of the leptophilic scenario introduced before.

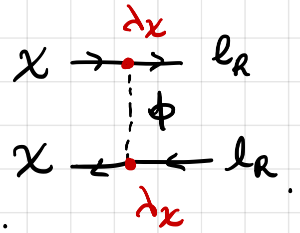
DM, self conjugate Majorana fermion

$$\mathcal{L} \supset \lambda_\chi \, \bar{\chi} \, l_R \, \phi + h.c.$$

↳ EM charged scalar.

DM annihilation:

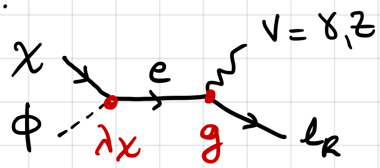
$$\sigma_{11} \propto \lambda_\chi^4$$



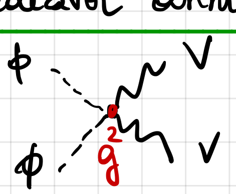
NB: see extra contribs on p47.

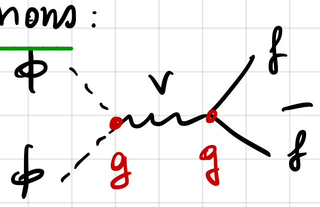
Co-annihilations, e.g:

$$\sigma_{12} \propto \lambda_\chi^2 \, g^2$$



Mediator annihilations:

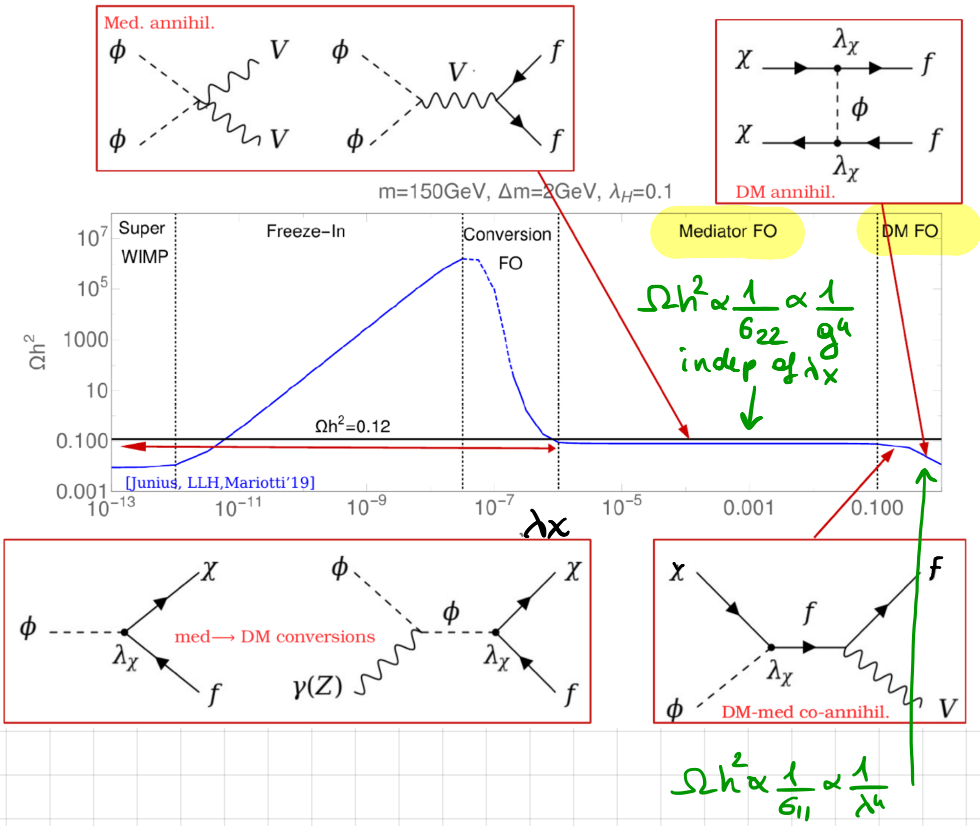




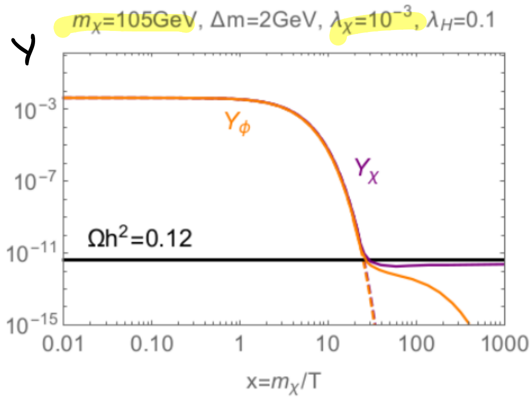
$$\sigma_{22} \propto g^4$$

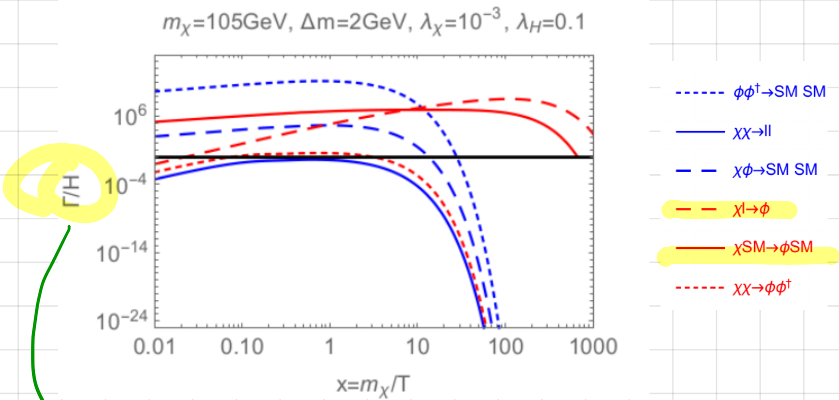
Depending on the $\Delta_{12} = \frac{m_\phi - m_\chi}{m_\chi}$ and the relative strength / # of channels involved in the annihilation / co-annihilation channels the DM relative abundance of 0.12

Can be obtained for $\lambda_\chi > 10^{-6}$ through the FO mechanism assuming <u>both</u> chemical <u>and</u> kinetic equ between DM-med and SM. → not really a "WIMP"



Mediator annihilation FO in the dephobic scenario (credit: Sam Junius)

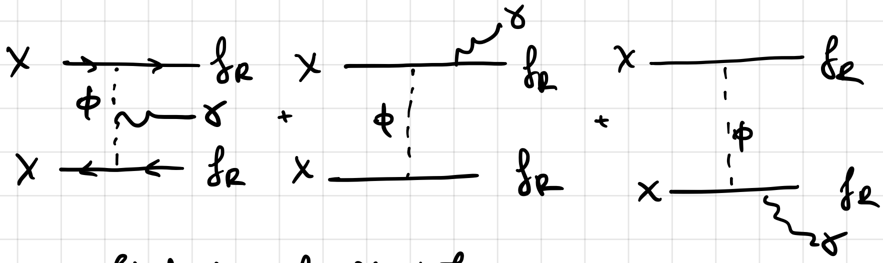




$$\Gamma = \frac{\gamma_{ij}}{n_i^{eq}}$$

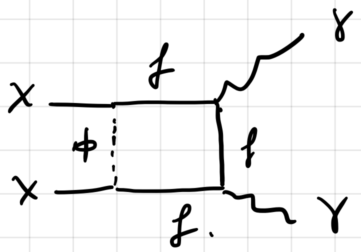
* an interesting note on the "t-channel" models is that you can have nice line-like features from:

• Virtual-internal-Bremsstrahlung.



which is large when
$m_f \ll m_\chi$ and $\Delta_{12} < 1$

• loops like eg.



⇝ relevant for indirect detection searches but also relic abundance if $f_R = q_R$. especially when χ is a self conjugate scalar field.

See eg. 1307.6480, 1511.04452, 1503.01500, ...

V FEEBLY INTERACTING MASSIVE PARTICLES

The case of feebly interacting massive particles (FIMPS) is usually directly associated to the Freeze-in mechanism of production.

Here I will refer to FIMPS as particles that interacts with the SM or the mediator of interactions with coupling much more feeble than weak interactions.

Within this framework I will describe 3 production mechanism:

MOV 1| the Freeze-in (Fi)
MOV 2| the superWIMP mechanism (SW)
~~DS~~ ~~MO~~ 3| the {conversion / coscattering} driven Freeze out. (conv F.O.)

- Going from FI to SW mechanisms, smaller couplings are involved.

- We can also go from
  $Fi \to conv FO \to$ co-annihilations FO $\to$ FO,
  by increasing the couplings but small couplings between the DM and a dark sector partner is needed.

Here we will work in a framework where:

- Z_2 odd particles: $\begin{cases} B = \text{both particle in thermal } \underline{\text{and}} \text{ chemical eq at early time} \\ \chi = \text{DM} \end{cases}$

- Production happens in a radiation dominated era i.e. $H(T_{prod}) \sim \frac{T^2}{M_P}$.

in the case of e.g FI:

- Notice that you can also produce DM directly from SM in models such as $\begin{cases} \text{dark photon mediated interactions} \\ \text{Higgs portal} \end{cases}$
  <u>or</u> with the mother particle out-of equilibrium (sequential FI).
  See e.g. 1908.09864, 2005.06294, 0911.1120.

- DM production in a modified early cosmology can have very interesting impact for FIMP detection at colliders and the interplay with cosmology
  <u>see</u> e.g. 2102.06221 for an early MD era

V.1. Freeze-in

Here we will deepen the core of FI from a mother particle decay into SM particle(s) and the DM : $B \to X$.

V.1.1. Rule of thumb

One can guess the typical dependence of Y_X in terms of Γ, m_B and M_P.

Indeed the amount of DM particles at a given time for a production rate R should be

$$Y_X \sim R \cdot t .$$

Considering $t \sim \frac{1}{H(T)} \sim \frac{M_P}{T^2}$

and $R = \underbrace{\Gamma_{B\to X}}_{\text{decay rate in the rest frame}} / \gamma \to$ Lorentz factor $\rightsquigarrow$ time dilation
$\Delta' t = \Delta t \cdot \gamma$
$\gamma = \frac{E_B}{m_B} \sim \frac{T}{m_B}$

$$\Rightarrow Y_X \sim \Gamma_{B\to X} \frac{m_B}{T} \frac{M_P}{T^2} \sim \frac{\Gamma_{B\to X} M_P x^3}{m_B^2}$$

$\rightsquigarrow$ the DM production is more efficient at low T $\to$ IR dominated process for FI through decays

Considering that production get to a halt when both particles gets Boltzmann suppressed at $x \simeq \mathcal{O}(1)$ $\rightarrow$ the lowest possible T is $T \simeq m_B$

$\Rightarrow$ We expect $\quad Y_\chi \sim \dfrac{\Gamma_{B\to\chi}\, M_P}{m_B^2}$.

V 1.2. Boltzmann equations.

In this case we can go back to.

$$\frac{df_\chi}{dt} = \frac{1}{E_\chi}\, C[f_\chi].$$

Assuming that there is no initial density of DM $n_\chi(t_i) = 0$, we can neglect the reverse process $\chi \to B$ and write.

$$\frac{1}{E_\chi} C[f_\chi] = \frac{1}{2E_\chi}\int \Pi_\alpha\left(\frac{d p_\alpha}{(2\pi)^3 2E_\alpha}\right)(2\pi)^4\delta^4(P_{fin}+p_\chi - P_{in})$$
$$|M|^2_{in\to fin+\chi}\; f_{in}\,(1\pm f_\chi)(1\pm f_{in})$$

Let us emphasize that in the case of FI it has been shown that spin-statistic can affect the results. Here however, as in the case of WIMPS, we will neglect the $(1\pm f)$ factors., see e.g. 1801.03508 & micrOMEGAs.

In the latter case, we can again integrate our both side of the equation over the DM 3 momenta and we obtain:

decay width $B \to X$

$$\frac{dn_X}{dt} + 3H n_X = n_B^{eq} \, \Gamma_{B\to X} \, \frac{K_1[m_B/T]}{K_2[m_B/T]}$$

$K_n \equiv$ Modified Bessel function of the second kind.

when considering N.R. bath particles, ie $f_B^{eq} = g_B \exp(-E_B/T)$

We also see that defining the time variable

$$\boxed{x = \frac{m_B}{T}}$$

is more convenient in the case of FI as the DM production will become exponentially suppressed as the bath particle becomes non relativistic. Here again going from time to temperature, using entropy conservation, we use

$$\frac{d\ln T}{dt} = -\bar{H} \quad \text{and}$$

$$\frac{dY_X}{dx} = \frac{\Gamma_{B\to X}}{x\bar{H}} \, Y_B^{eq} \, \frac{K_1[x]}{K_2[x]}$$

Assuming constant g_* and h_{eff} over DM production, one gets

$$Y_X^\infty = \frac{405\sqrt{5}}{4\sqrt{2}\,\pi^4}\;\frac{g_B}{h_{eff}\, g_*^{1/2}}\;\underbrace{\frac{\Gamma_{B\to X}\, M_P}{m_B^2}}_{}.$$

we recover the rule of thumb dependence.

$$\to \Omega_X h^2 = \frac{Y_X^\infty\, s_0\, m_X}{\rho_c/h^2}$$

$$\simeq 0.12\left(\frac{m_X}{10\,\text{keV}}\right)\left(\frac{1\,\text{TeV}}{m_B}\right)^2\left(\frac{g_B\,\Gamma_{B\to X}}{5\,10^{-15}\,\text{GeV}}\right)$$

here we use $g_* = h_{eff} \simeq 100$

Considering the case of e.g $m_B \gg m_X, m_{SM}$ in $B \to X\, SM$, one has

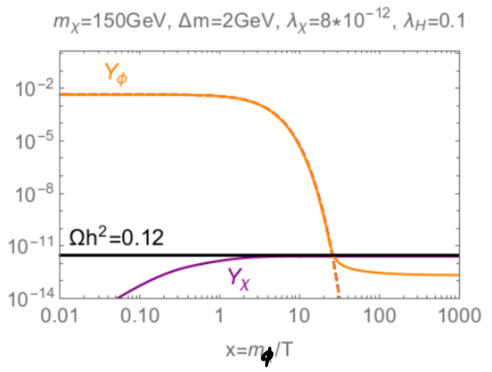
$$\Gamma_{B\to X} = \frac{\lambda_X^2}{8\pi}\, m_B \quad \text{ie.} \quad \lambda_X \sim 8\, 10^{-9} \ll g$$

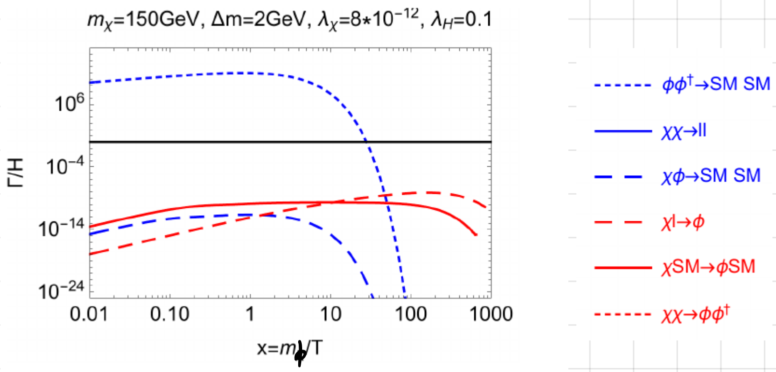
$SU(2)_L$ coupl.

In order to account for all DM, ie, the DM is <u>much more feebly coupled</u> than in the case of WIMP.

VI.3 Illustration.

Below some illustrations in the case of leptophilic DM.

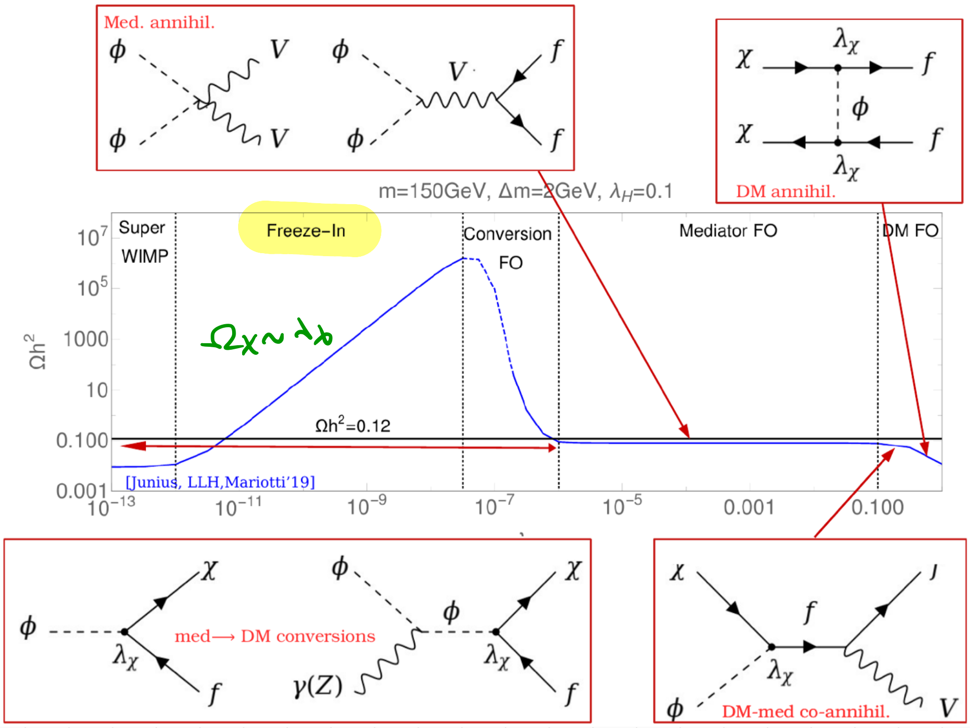




All processes except for mediator annihilation are slow compared to Hubble rate.

For the set of masses below $m_\chi, m_\phi \sim 100$ GeV we produce DM through FI for λ_χ between 10^{-12} and 10^{-8}

$\Omega_\chi h^2 \sim \Gamma_{B \to \chi} \sim \lambda_\chi$



V 2. The Super WIMP (SW) mechanism

Considering even lower coupling for DM - mediator - SM, we end up with very low decay rate $\Leftrightarrow$ long life time of the bath particle. Very few DM would be produced at early time through FI

- In the latter case, the bath particle could have gone through Freeze-out. and

$$Y_B = Y_B^{FO} \quad \text{for} \quad x_f^B < x < x_{SW}.$$

At latter time, the B-particle decays fully to DM. We can thus expect

$$Y_B^{FO} = Y_X^{\infty} \quad \Longrightarrow \quad \boxed{\Omega_X^{SW} h^2 = \frac{m_X}{m_B} \Omega_B h^2}$$

- We can recover $\Omega_X h^2$ from the same Boltzman eqs. as in the case of FI simply using the ansatz

$$f_{in} = f_B = \frac{f_B^{eq}}{Y_B^{eq}} Y_B$$

→ same ansatz as for vanilla WIMP & Boltzmann. see pp. 35, 42

This ansatz is valid while B is in kinetic equilibrium. After FO, the B Boltzmann equ. reduces to:

$$\frac{dY_B}{dx} = -\underbrace{\frac{\Gamma_{B\to X}}{H x}}_{\propto \frac{\Gamma_{B\to X} M_0}{m_B^2}} Y_B \underbrace{\frac{K_1(x)}{K_2(x)}}_{\substack{x > x_f \text{ ie } x \gg 1 \\ K_1[x]/K_2[x] \to 1}} \qquad x > x_{FO}^B$$

with $Y_B = Y_B^{FO} = cst.$ at $x = x_{FO}^B$, and $g_* = cst$

$$\Rightarrow \boxed{Y_B(x) = Y_B^{FO} \, e^{-R_\Gamma (x^2 - x_{FO}^2)/2}} \qquad x \overset{B}{>} x_{FO},$$

with $R_\Gamma = \frac{\Gamma_{B\to X} M_0}{m_B^2}$; $M_0 = \sqrt{\frac{45}{4\pi^3 g_*}} \times M_p$

⇒ the decay time or x_{SW}, for $x_{SW} \gg x_{FO}$ is:

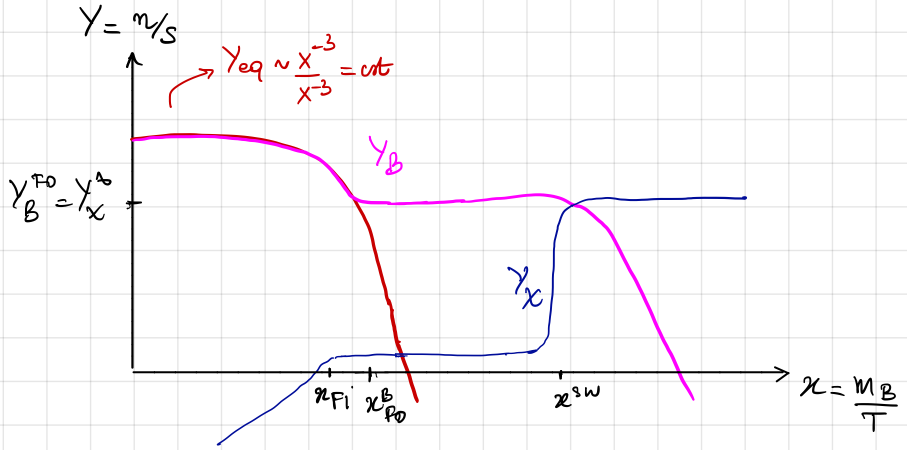
$$\boxed{x_{SW}^2 = \left(\frac{R_\Gamma}{2}\right)^{-1}}$$

Then going back to x equation

$$\Rightarrow \frac{dY_X}{dx} = \frac{\Gamma_{B\to X}}{x\,\bar{H}} \cancel{\frac{Y_B^{eq}}{Y_B^{eq}}} Y_B^{FO} \underbrace{\frac{K_1[x]}{K_2[x]}}_{\sim 1 \text{ for } x \gg 1}$$

$$\Rightarrow Y_X(x_{SW}) = Y_B^{FO} \cdot R_\Gamma \int_0^{x_{SW}} dx\, x = Y_B^{FO} \cdot$$

as expected !



- As a final comment as

$$Y_X^\infty = Y_B^{FO} \Rightarrow \Omega_X^{SW} h^2 \propto Y_B^{FO} \propto \frac{1}{\sigma_{22}} \propto \frac{1}{g^2}$$

which is independent of λ_X

V 3. Conversion driven F.O.

in IV we have seen that we can go from the vanilla WIMP case involving couplings $\sim g_{SU(2)_L}$ to the case where the DM would get its relic abundance fixed by the Z_2 odd partner if the relative mass splitting is small.

Now for 3 body interaction $\mathcal{L} \supset \lambda_\chi B \chi A_{SM}$

the dependence $\left(\Delta = (m_\phi - m_\chi)/m_\chi \right)$

$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma v_{eff} \rangle} \propto \begin{cases} \lambda_\chi^{-4} & \text{DM FO} \\ \lambda_\chi^{-2} g^{-2} \, 1/\exp(-\Delta x) & \text{CO-ANN FO} \\ g^{-4} \, 1/\exp(-2\Delta x) & \text{MEDIATOR FO} \end{cases}$$

assumes that $B \leftrightarrow \chi$ conversion processes happen fast enough to ensure chemical equilibrium and as a result

$$\frac{n_B}{n_B^{eq}} = \frac{n_\chi}{n_\chi^{eq}}$$

It has however been noticed though, see 1705.09292, 1705.084600, that when conversion processes get suppressed enough so that $\frac{\gamma_{i \leftrightarrow j}}{H\, m_i} < 1$ where $\gamma_{i \leftrightarrow j}$ is the reaction rate associated to $\chi_i \leftrightarrow \chi_j$ conversions, the departure from chemical equilibrium re-introduce a λ_χ dependence in the $\Omega_\chi h^2$ for $\lambda_\chi \ll 1$ and $\Delta_{ij} < 1$.

In order to compute correctly the DM abundance, one should definitively account for $\gamma_{i \leftrightarrow j}$ effects on the $\{Y_i\}_{i=1..N}$

For the framework we are concerned with considering $\chi_1 = \chi$ and $\chi_2 = B$, we should solve the coupled Boltzmann system:

$$\frac{dY_1}{dx} = \frac{-1}{Hx}\left[\gamma_{11}\left(\frac{Y_1^2}{Y_{1eq}^2} - 1\right) + \gamma_{12}\left(\frac{Y_1 Y_2}{Y_{1eq} Y_{2eq}} - 1\right) \boxed{- \gamma_{2\to 1}\left(\frac{Y_2}{Y_{2eq}} - \frac{Y_1}{Y_1^{eq}}\right)} + \gamma_{11\to 22}\left(\frac{Y_1^2}{Y_{1eq}^2} - \frac{Y_2^2}{Y_{2,eq}^2}\right)\right]$$

$$\frac{dY_2}{dx} = \frac{-1}{Hx}\left[\gamma_{22}\left(\frac{Y_1^2}{Y_{1eq}^2} - 1\right) + \gamma_{12}\left(\frac{Y_1 Y_2}{Y_{1eq} Y_{2eq}} - 1\right) \boxed{+ \gamma_{2\to 1}\left(\frac{Y_2}{Y_{2eq}} - \frac{Y_1}{Y_1^{eq}}\right)} - \gamma_{11\to 22}\left(\frac{Y_1^2}{Y_{1eq}^2} - \frac{Y_2^2}{Y_{2,eq}^2}\right)\right]$$

<u>Caution</u> : in order to write the above equations, we are assuming that χ is in kinetic equ. with B. This can not be ensured for arbitrarily small couplings!

It has been shown that for the parameter space testable by experiments with a B ≡ colored particle. kinetic equilibrium is a good approach, see 1705.09292. In some other cases, where the coupling involved in coscattering processes is smaller, it is necessary to go back to the unintegrated Boltzmann eqs. see e.g. 1705.08450.

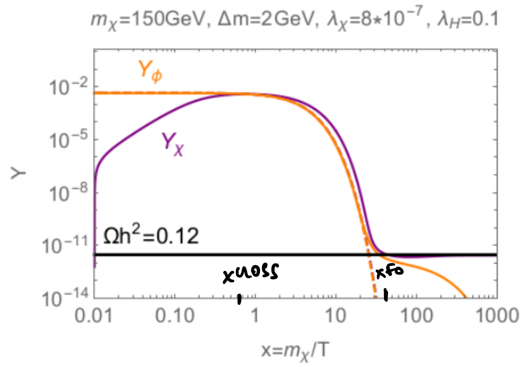
Here, we describe the case where departure from kinetic equilibrium can be neglected and the use of the integrated Boltzmann equ. can be trusted*

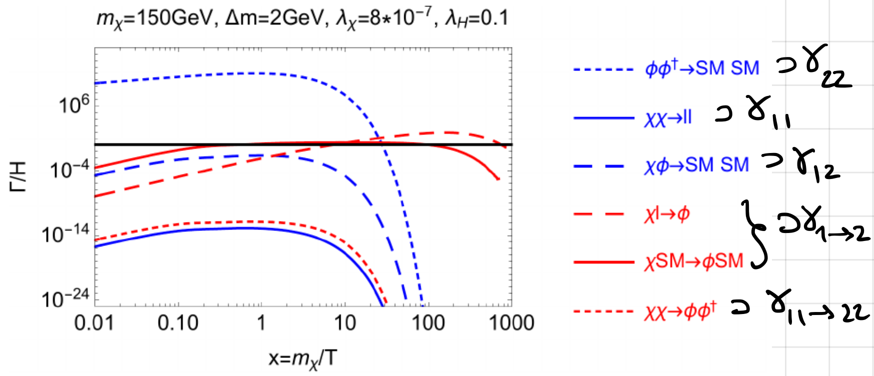
* in the $(\lambda_\chi, \Omega_\chi h^2)$ plot for leptophilic DM, we have used a dashed line to emphasize the region where we do not expect kinetic eq.

In the framework of conversion driven F.O. we can not provide analytic results but numerical results.

We have thus to discuss a specific model, the leptophilic DM case: $\mathcal{L} \supset \lambda_\chi \bar{\chi} e_R \phi$

Below, we show the DM and mediator density evolution for $m_{DM} = 150$ GeV, $\Delta m_{\chi\phi} = 2$ GeV and $\lambda_\chi = 8 \cdot 10^{-7}$





Taking $Y_\chi(x=0)=0$ as initial condition $Y_\chi(x)$ slowly builds up as in the case of FI. This time though $Y_\chi(x)$ is going to cross $Y_\chi^{eq}(x)$ before it freezes-out.

In contrast, ϕ is maintained in chemical equie with the plasma thanks to its gauge interactions.
On the other hand $\chi \to \phi$ conversions are barely efficient as can be seen in the bottom plot with red colors:

$$\Gamma_{\chi\to\phi} = \frac{\gamma_{\chi\to\phi}}{n_\chi^{eq}} \lesssim H$$

Such barely efficient conversion processes maintain : $Y_\chi(x) \gtrsim Y_\chi^{eq}(x)$ $x_{cross} < x < x_{FO}$

As a result, once the mediator gets out of chemical equilibrium at $x_{FO}^B \simeq 25$ and eventually decays to χ, the DM density freezes out at slighly later time.

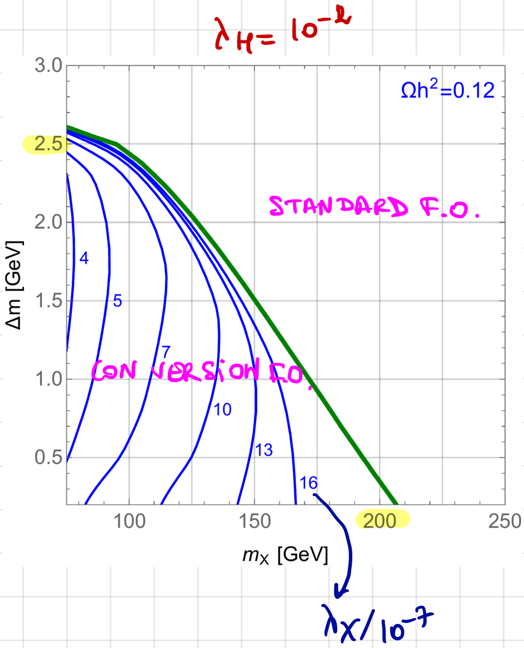
Now, the smaller is λ_χ, the more important will be the departure of $Y_\chi(x)$ compared to Y_χ and the larger $Y_\chi(x_f)$ becomes.

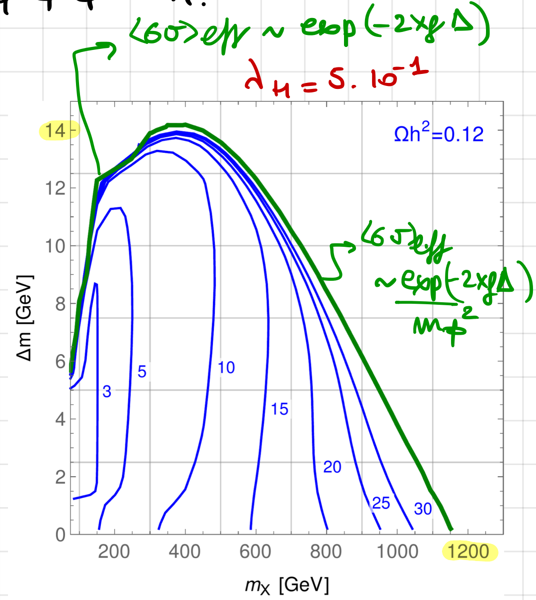
This behavior of $\left\{ \begin{array}{l} Y_\chi \nearrow \\ \Omega_\chi h^2 \end{array} \right.$ for $\lambda_\chi \searrow$ is well visible in our $(\lambda_\chi, \Omega_\chi h^2)$ plot.

One last comment. If we consider additional $BB \to SMSM$ process, we can actually decrease the expected $\Omega_X h^2$ that would be expected from mediator annihilation. This also implies that a larger $(m_B, \Delta m)$ parameter space is able to give rise to conversion driven F.O.

The reason for this is because if $\sigma_{22} \uparrow \Rightarrow \Omega_X h^2|_{\substack{med\\ann}} \downarrow$
$\Rightarrow$ more parameter space to compensate with λ_X due to inefficient conversions.

In the case of dephophic DM, an additional annihilation channel is provided by considering $\mathcal{L} \supset \lambda_H \phi^\dagger \phi H^\dagger H$.





Green limit of the parameter space corresponds to $\Omega_\chi h^2 = 0.12$ due to mediator annihilation F.O. i.e. for
$\langle\sigma v_{eff}\rangle \propto \langle\sigma v_{22}\rangle \exp(-2x_f\Delta)$. with $\Delta = \frac{\Delta m_{\chi\phi}}{m_\chi}$

The form of the latter can be easily understood as a competition between the mediator annihilation cross section $\langle\sigma v\rangle_{22}$ and the Boltzmann suppression factor $\exp(-2x_f\Delta)$, see the plot.

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lecture notes M. Tytgat IPM school 2015