AbsSynthe: abstract synthesis from succinct safety specifications

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Outline

1 Succinct safety specs = Safety games

2 The classic algorithm

- Main idea
- The uncontrollable predecessors' operator

3 A CEGAR algorithm

- Contributions
- Abstract game
- Abstract operators
- The algorithm

4 Benchmarks & conclusions



In essence: a boolean network for a single-output sequential circuit:

- A set of boolean inputs X,
- a set of boolean latches L with a distinguished error latch $BAD \in L$.



The circuit defines a boolean function f_l over L and X per latch.



For synthesis, X partitioned into uncontrollable X_u and controllable inputs X_c . X_u are chosen by the environment.





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• Q is the set of valuations of $L, U \subseteq Q$ are the error states

- Σ_u, Σ_c are valuations of X_u, X_c resp.
- $\delta: Q \times \Sigma_u \times \Sigma_c \to Q$ defined by circuit $\mathcal C$
- Game: environment chooses σ and controller responds with τ

$$\rightarrow$$
 $(0,0)$ $(1,0)$ $(1,1)$

Example: $L = \{l_0, BAD\}$, $\Sigma_u = \{r, \overline{r}\}$, $\Sigma_c = \{g, \overline{g}\}$.



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Based on the Reachability Game played on the graph $\langle Q, \Sigma_u, \Sigma_c, \delta, \mathcal{U} \rangle$:

- Define an uncontrollable predecessors operator UPRE.
- Compute the least fixpoint of UPRE starting from the error states (call this W_u).
- Section W_c = Q \ W_u controller can respond to a given σ with any τ which ensures she stays in W_c.



UPRE(S) is the set of states from which environment can force to reach S If $\Sigma_u = \{\sigma_0, \overline{\sigma_0}\}$ and $\Sigma_c = \{\tau_0, \overline{\tau_0}\}$, then UPRE(d, e) =???





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- Define an uncontrollable predecessors operator UPRE.
- Compute the least fixpoint of UPRE starting from the error states (call this W_u).
- So From $W_c = Q \setminus W_u$ controller can respond to a given σ with any τ which ensures she stays in W_c .



How do we compute UPRE?

Using BDDs...

Either compute a transition relation

$$T(L, X_u, X_c, L') = \bigwedge_{l \in L} l' \Leftrightarrow f_l(X_u, X_c, L)$$

and then set $\mathsf{UPRE}(S) = \exists X_u, \forall X_c, \exists L' : T(L, X_u, X_c, L') \land S(L');$ or

② for deterministic systems we can avoid computing T and just substitute f_l for each l in S^1

$$\mathsf{UPRE}(S) = \exists X_u, \forall X_c : S(L')[I' \leftarrow f_I(X_u, X_c, L)]_{I \in L}.$$



¹[Coudert et al., 1990, Coudert et al., 1991]

Brenguier, Pérez, Raskin, Sankur (ULB)

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Use information from the computation of the over-approx of UPRE to

- over-approx reachable states fixing winning strategies of environment
- restrict the uncontrollable actions we need to check to compute UPRE.
- Use substitution (BDD composition) to avoid computing an abstract transition relation (though post over-approx'd).
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$$L = \{l_0, l_1, l_{BAD}\}$$
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• Q^a defined by predicates $p_U = l_{BAD}, p_I = \neg (l_0 \lor l_1 \lor l_{BAD}), p_0 = l_0$



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- Q^a defined by predicates $p_U = l_{BAD}, p_I = \neg (l_0 \lor l_1 \lor l_{BAD}), p_0 = l_0$
- Δ^a over-approximates δ



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Some remarks:

- We require the initial state be distinguishable and \mathcal{U}^a to contain \mathcal{U} .
- The partition of Q is done (mainly) via localization reduction (only p_R, p_I, p_U are real predicates).



P is set of predicates defining Q^a . T^a is computed as expected from T.

Definition (Two UPRE operators)

Given $S^a \subseteq Q^a$ let

•
$$\overline{\mathsf{UPRE}}_a(S^a) = \exists X_u, \forall X_c, \exists P': T^a(P, X_u, X_c, P') \land S^a(P'),$$

• UPRE_a(S^a) =
$$\exists X_u, \forall X_c, \forall P' : T^a(P, X_u, X_c, P') \Rightarrow S^a(P')$$

In fact, one can again avoid computing T^a using substitution.

Lemma (Over- and under-approximating UPRE)

 $\gamma(\underline{\textit{UPRE}}^*_{a}(\mathcal{U}^{a})) \subseteq \textit{UPRE}^*(\mathcal{U}) \subseteq \gamma(\overline{\textit{UPRE}}^*_{a}(\mathcal{U}^{a})).$



Abstract UPRE: definition by example

If
$$\Sigma_u = \{\sigma_0, \overline{\sigma_0}\}$$
 and $\Sigma_c = \{\tau_0, \overline{\tau_0}\}$, then

• $\overline{\mathsf{UPRE}}_a(\{\{d, e\}\}) = ???$





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A CEGAR algorithm [de Alfaro and Roy, 2010]

Based on the abstract game $\langle Q^a, q_l^a, \Sigma_u, \Sigma_c, \Delta^a, \mathcal{U}^a \rangle$ and an over-approximation of the reachable states R^a :

- If $q_I^a \in \underline{\text{UPRE}}^*_a(\mathcal{U}^a)$ environment wins,
- ② if $q_I^a \notin \overline{\text{UPRE}}_a^*(\mathcal{U}^a)$ controller wins,
- else we do not know who wins G...add a new "useful" single-latch predicate to P and repeat.

Does it terminate?

Eventually all latches are added, so we converge to the original game.



Assume $q_I^a \notin \underline{\mathsf{UPRE}}^*_a(\mathcal{U}^a)$ and $q_I^a \in \overline{\mathsf{UPRE}}^*_a(\mathcal{U}^a)...$

- Extract a winning non-deterministic strategy of environment $\Lambda^a : Q^a \to \mathcal{P}(\Sigma_u)$,
- ② this defines a non-det strategy for him in the original game $\Lambda : Q \rightarrow \mathcal{P}(\Sigma_u).$

Theorem (All of his winning strats)

If λ is a winning strategy for environment in G, then λ is "included" in Λ .



This allows for two nice optimizations!

Corollary (Over-approx reachable and restrict UPRE)

- If $q_I^a \in \overline{UPRE}^*_a(\mathcal{U}^a)$ then we can restrict our search to states reachable if environment plays $\Lambda^a(P, X_u)$,
- and we can replace UPRE by

$UPRE_{\Lambda}(S) = \exists X_{u}, \forall X_{c}, \exists L': T(L, X_{u}, X_{c}, L') \land \Lambda(L, X_{u}) \land S(L')$

which takes less uncontrollable inputs into account.



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$\mu X.(\mathcal{U}^{a} \cup \overline{\mathsf{UPRE}}_{a}(X))$

- Look for a strategy of environment in the abstract game.
- Ignore all states not reachable in original game via these strategies.



 $\mu X.(\mathcal{U}^a \cup \overline{\mathsf{UPRE}}_a(X)) \cap R^a$

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We don't have a unique answer :-(

Definition (Interesting and useful latches)

Given \mathcal{U}^a and current visible latches,

- **(**) we consider a latch *I* interesting if $I \not\Rightarrow U^a$ and $\neg I \not\Rightarrow U^a$; and
- 2 we say an interesting latch is useful if there is some already visible latch v such that $f_v(L, X_u, X_c)$ depends on I.

The idea is. . .

The newly visible latch will hopefully make Δ^a more closely resemble the original δ .



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$abs_synth(G, G^a, R^a)$

1
$$w_u := \mu X. (\mathcal{U}^a \cup \underline{\mathsf{UPRE}}_a(X)) \cap R^a;$$

2 if $q_l^a \in w_u$ then return not controllable;
3 $prev := \emptyset;$
4 while $R^a \neq prev$ do
5 $| prev := R^a;$
6 $| W_u := \mu X. (w_u \cup \overline{\mathsf{UPRE}}_a(X)) \cap R^a;$
7 $| if q_l^a \notin W_u$ then return controllable;
8 $| \Lambda^{env} := \text{non-det strategy defined by } (w_u);$
9 $| R^a := \mu X. (q_l^a \cup \text{post}(X, \Lambda^{env})) \cap R^a;$
10 end
11 $w'_u := (\mathsf{UPRE}_{\gamma(\Lambda^{env})}(\gamma(w_u))) \cap \gamma(R^a);$
12 if $w'_u \subseteq \gamma(w_u)$ then return controllable;
13 $Q_2^a := \text{refine}(Q^a, w'_u \cup \gamma(w_u), \gamma(R^a));$
14 $\mathcal{U}_2^a := \underline{\alpha}_2(w'_u \cup \gamma(w_u));$
15 return $abs_synth(G, G_2^a, \overline{\alpha}_2(\gamma(R^a)));$

$abs_synth(G, G^a, R^a)$

Is q₁^a already in the FP of under-approx'd UPRE?

- 1 $w_u := \mu X. (\mathcal{U}^a \cup \underline{\mathsf{UPRE}}_a(X)) \cap R^a;$
- 2 if $q_I^a \in w_u$ then return not controllable;
- 3 prev := \emptyset ;
- 4 while $R^a \neq prev$ do
- 5 *prev* := R^a ;
- $\mathbf{6} \quad W_u := \mu X. \ (w_u \cup \overline{\mathsf{UPRE}}_a(X)) \cap R^a;$
- 7 **if** $q_I^a \notin W_u$ **then return** controllable;
- 8 $\Lambda^{env} :=$ non-det strategy defined by (w_u) ;

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1 $w_{\mu} := \mu X. (\mathcal{U}^a \cup \mathsf{UPRE}_a(X)) \cap R^a$ Is q_1^a not in the FP of 2 if $q_{I}^{a} \in w_{II}$ then return not contr over-approx'd UPRE? 3 prev := \emptyset ; If it is, just update 4 while $R^a \neq prev$ do reachability information prev := R^a ; 5 $W_{\mu} := \mu X. (w_{\mu} \cup \mathsf{UPRE}_{a}(X)) \cap R^{a};$ 6 **if** $q_i^a \notin W_u$ **then return** controllable; 7 Λ^{env} := non-det strategy defined by (w_{μ}) ; 8 $R^a := \mu X. (q^a \cup \text{post}(X, \Lambda^{env})) \cap R^a$; 9 10 end 11 $w'_{\mu} := (\mathsf{UPRE}_{\gamma(\Lambda^{env})}(\gamma(w_{\mu}))) \cap \gamma(R^{a});$ 12 if $w'_{\mu} \subseteq \gamma(w_{\mu})$ then return controllable; 13 $Q_2^a := \operatorname{refine}(Q^a, w'_{\mu} \cup \gamma(w_{\mu}), \gamma(R^a));$ 14 $\mathcal{U}_2^a := \alpha_2(w'_{\mu} \cup \gamma(w_{\mu}));$ 15 return abs_synth($G, G_2^a, \overline{\alpha}_2(\gamma(R^a))$);

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Some results



Figure : Time (in seconds) to check realizability.

Figure : Time (in seconds) for *cnt* benchmarks.

- (C) FP computation with a precomputed transition relation²;
- (C-TL) no transition relation;
- (A) CEGAR algo with a precomputed abstract transition relation;
- (A-TL) no transition relation (post overapproximated).

²Base implementation from [Bloem et al., 2014]

Thank you for your attention!

If you want to drink download our tool:

https://github.com/gaperez64/AbsSynthe



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