Verification of Run Graphs

(Some Preliminary Results)

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Distributed Systems

"A distributed system consists of a collection of distinct processes which are spatially separated & communicate with one another by exchanging messages."

- e.g., Apps based on Berkeley Socket API / MPI
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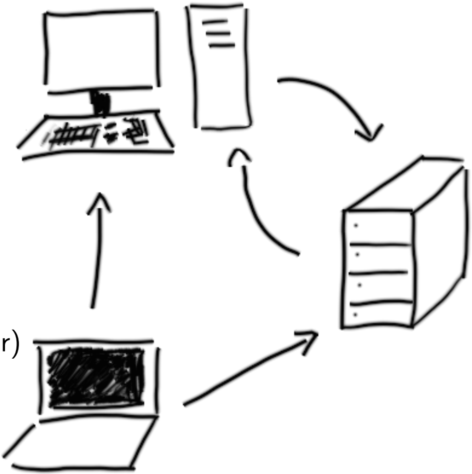
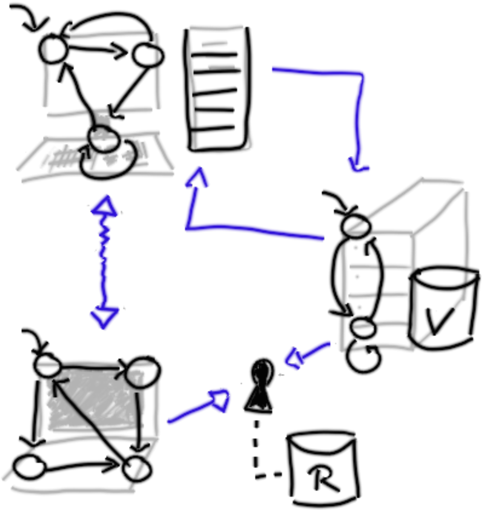


(Q)ueueing (C)ommunicating (P)rocesses

- processes modeled as local labeled transition system with "(infinite) data"
  - pushdown stack
  - variables
  - ...
- add synchronization via
  - reliable, unbounded, fifo queues
  - locks for shared resources
  - (classical) rendez-vous
  - barriers
  - ...



Recap: Ongoing Research

- fifo queues (I): Message Sequence Charts MSC
  - bounded branching of unfolding [Madhusudan '01]
  - ∃/∀ bounded channels [Lohrey/Musholl '04]
  - ...

semantic restrictions!

- fifo queues (II) [La Torre/Madhusudan/Parlato '08]
  - assert channels can only receive when local stack empty
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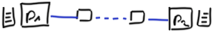
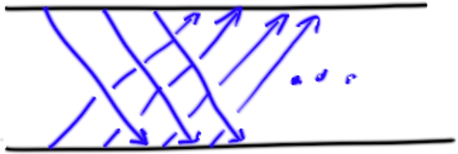
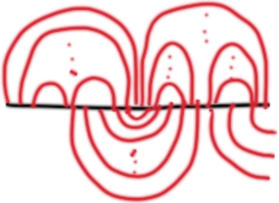
  - communication architecture does not allow 

Antipatterns/“Contra”-Patterns

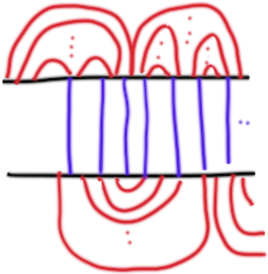


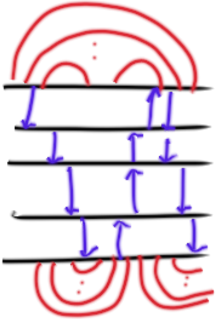
[Brand/Zafiropoulo ’83]

- distinctive causal entanglement patterns lead to undecidability issues

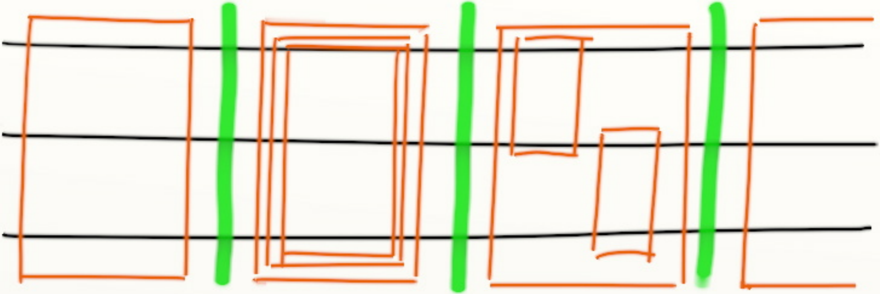


Multi-stack PDA





Decidability/"Pro"-Patterns



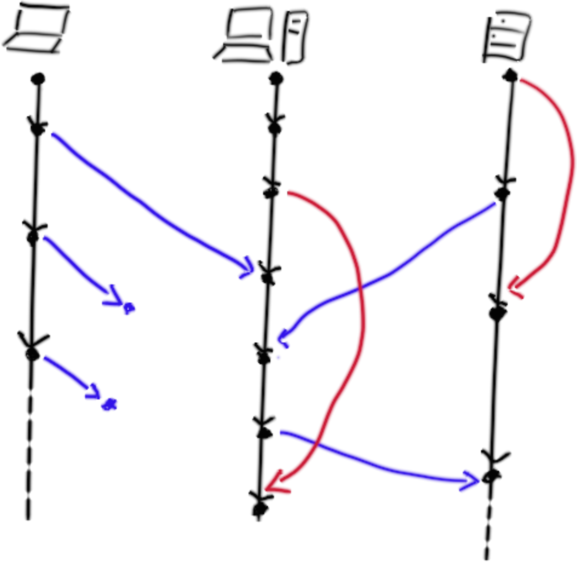
- cut run into sequence of sub-patterns
- only "exchange" finite information between these patterns
- sub-patterns either use recursion or composition of sub-sub-patterns
- "context-freeness"

How to Bridge The Gap ?

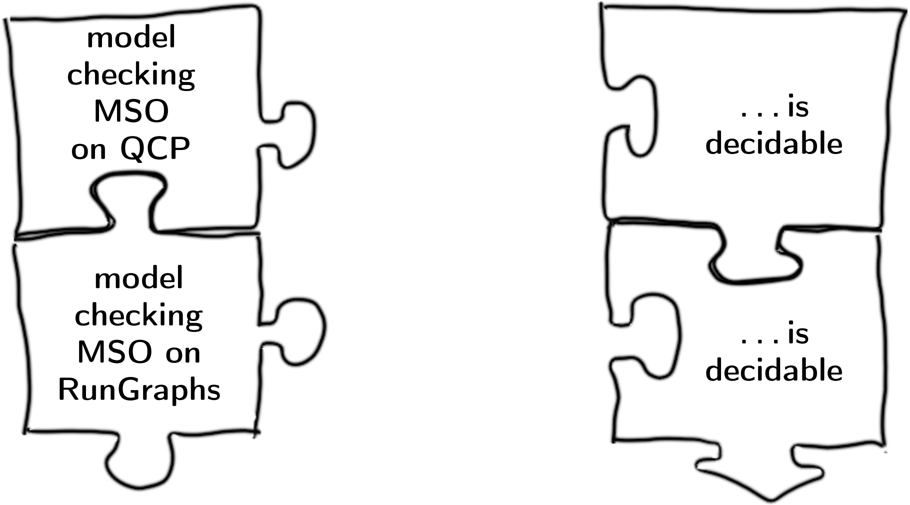
Run Graphs[1]

- each process defines a local total order of events
- additional causal constraints: synchronization (fifo) local data (push/pop)

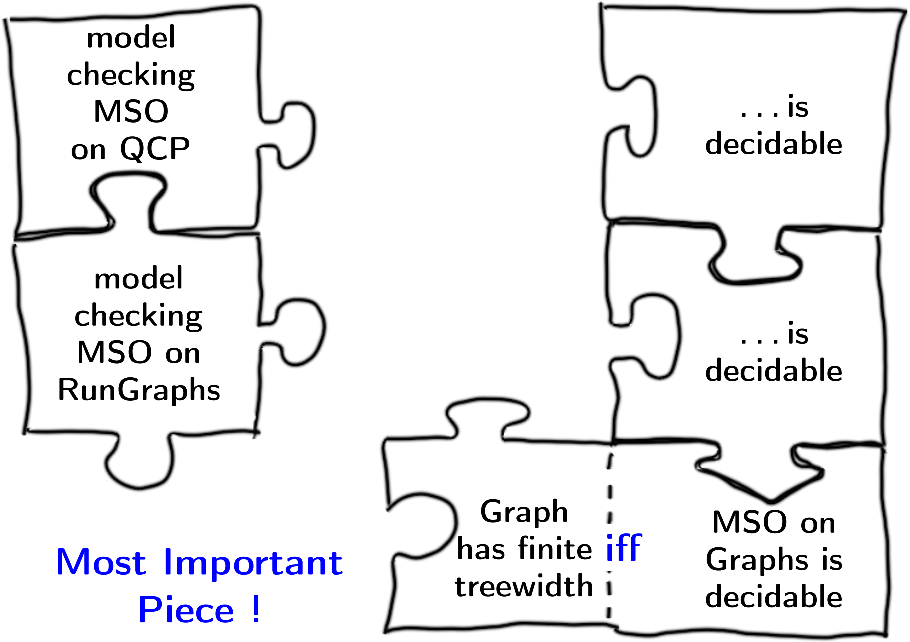
hence,
partial order semantics !



[1] aka "space-time diagrams", extended Hasse diagrams, MSC with pushdowns,. . .



“Easier” & more Concise Question, but still. . .


model checking MSO on QCP
model checking MSO on RunGraphs
Most Important Piece !
. . . is decidable
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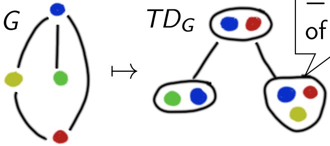
Graph has finite treewidth
iff
MSO on Graphs is decidable

Standing on the Shoulder of Giant Results

- treewidth measures how close a graph is to a tree
- classical way to calculate: tree decomposition

treewidth $=$ max size of "bag" -1



x

- apply (some) tree-based algos to general graphs via TD_G
- extension to classes of graphs leads to unbounded treewidth

[Courcelle's Theorem]

SAT(MSO) is fixed parameter tractable for given treewidth of model and size of formula.

[Madhusudan/Parlato '10(?)] based on [Seese '91]

Given class $\mathcal{C}$ of graphs of bounded treewidth which are MSO definable, and an MSO formula φ, then we can test whether there exists $G \in \mathcal{C}$ s.t. $G \models \varphi$.

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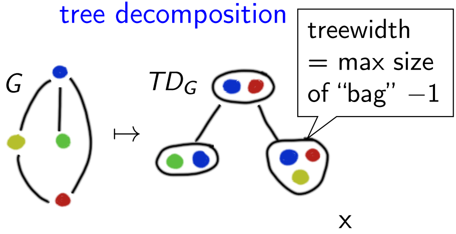

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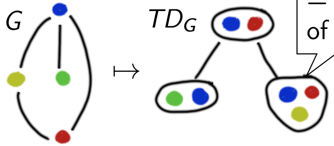

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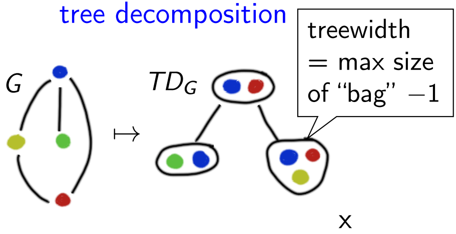

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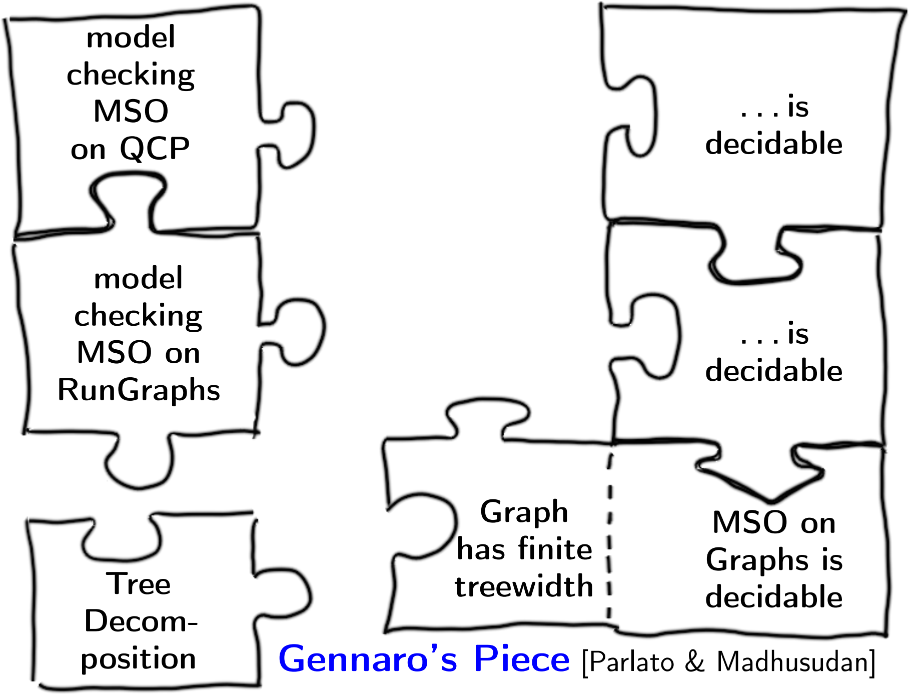
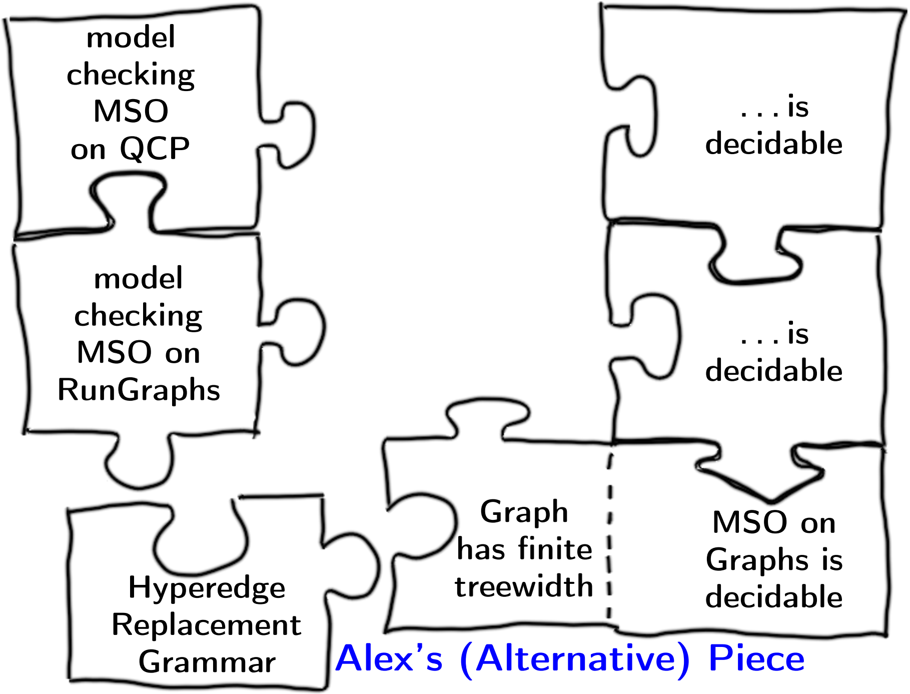
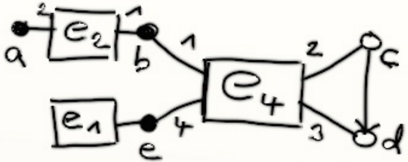


Gennaro's Piece [Parlato & Madhusudan]



Alex's (Alternative) Piece

Hyperedge Replacement Grammars



- *Hypergraph* $\mathcal{H} = \langle V, E, Ext \rangle$
  - vertices V, edges $E \in V^+$
  - external vertices $Ext \subseteq V$
- HR-grammar $\mathcal{G} = \langle N, T, \mathcal{R}, n^\bullet \rangle$
  - non-/terminals N/T
  - initial $n^\bullet \in N$
  - rules $\mathcal{R}$
- rule $R : X \in N \hookrightarrow \mathcal{H}$
  - $\mathcal{H}$ is hypergraph whose
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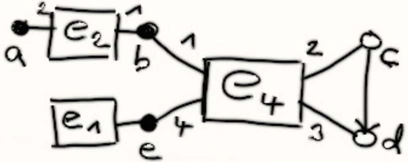

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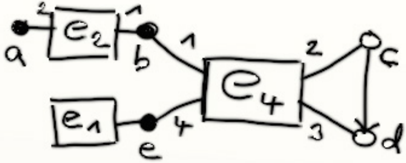

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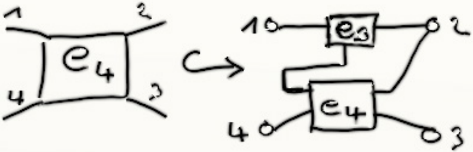

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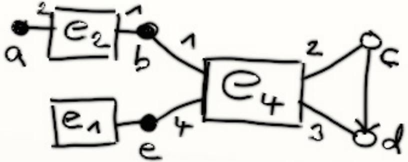

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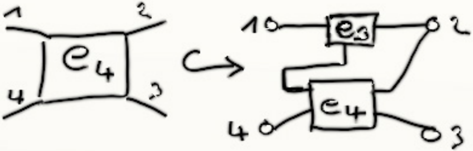

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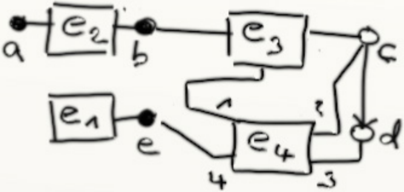

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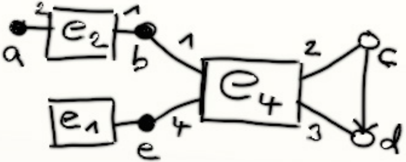

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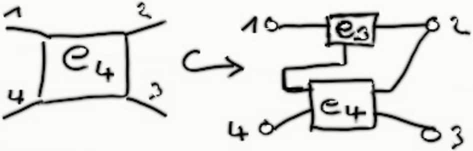

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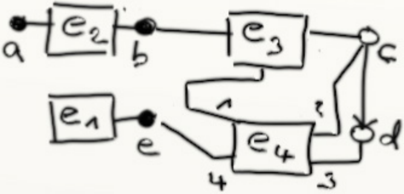

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[Seese '91]

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[Lautemann '88]

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➪ directly show undecidability

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example: MSC / CFSM

➪ easily finding positive results

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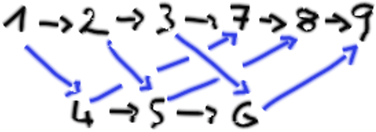

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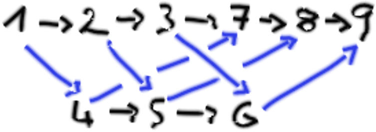

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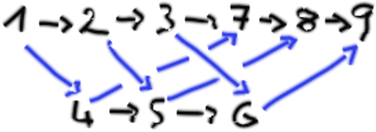

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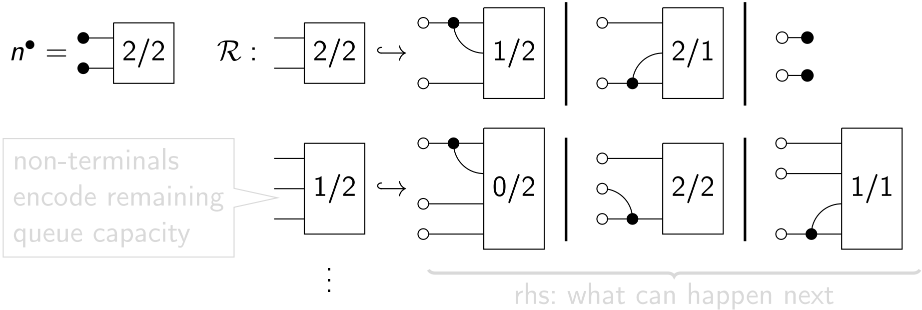

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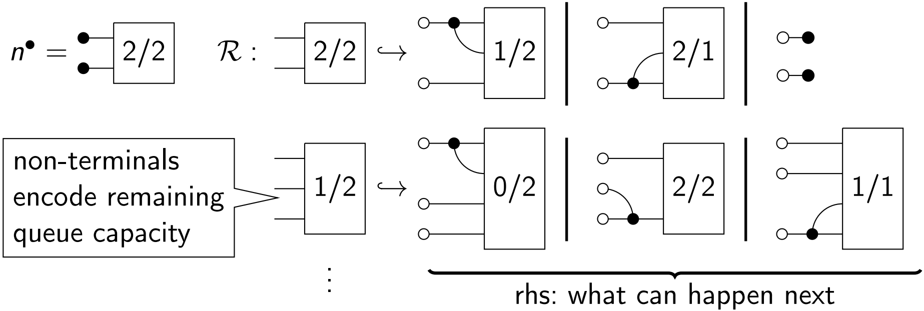

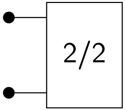

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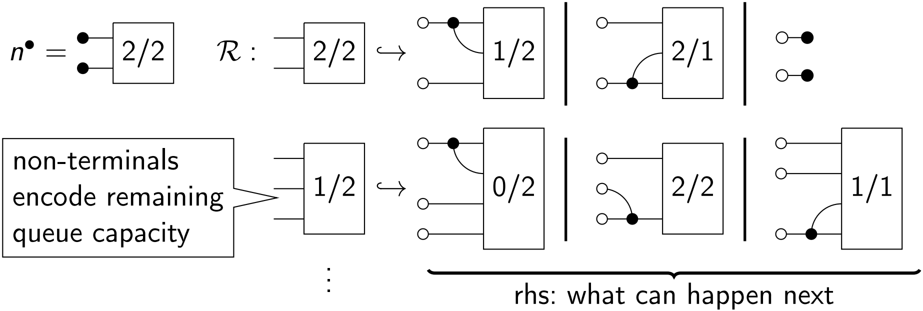

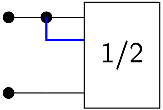

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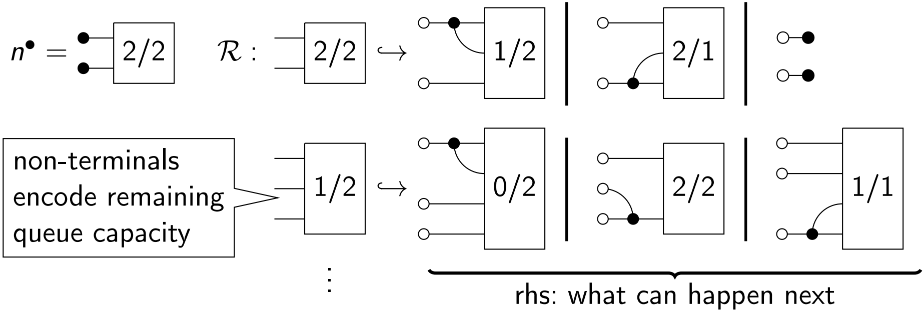

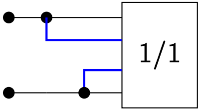

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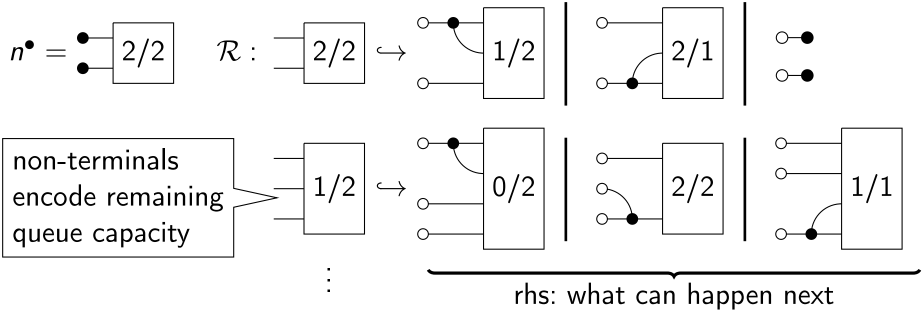

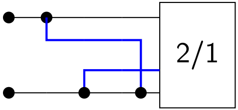

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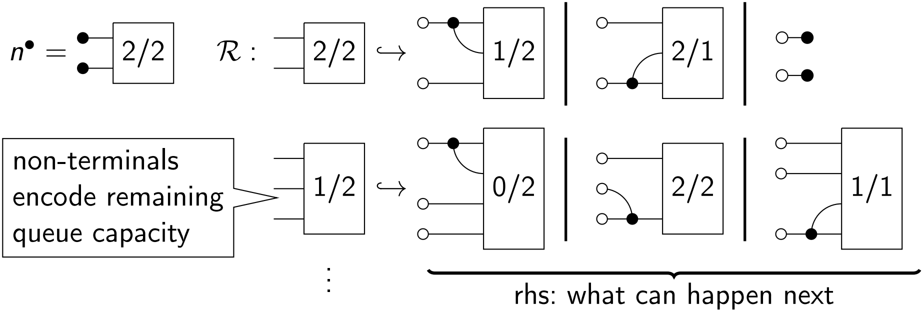

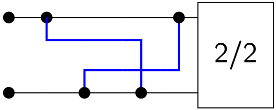

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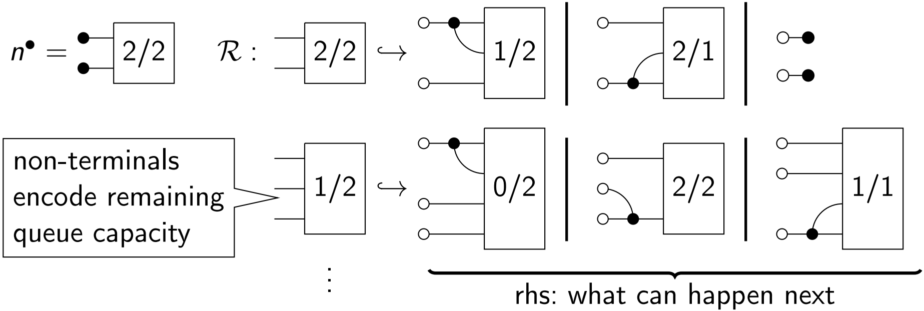

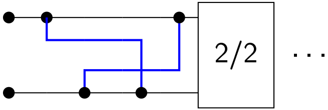

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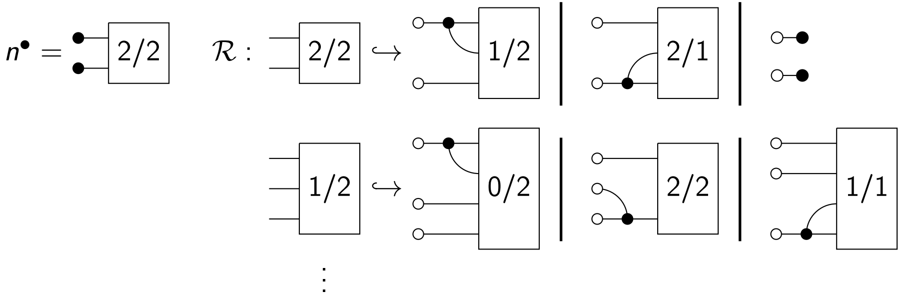
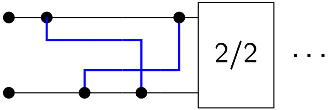




- HRG generates run graphs of 2-bounded MSC with 2 processes
- generalize: p processes and bound k
- treewidth of run graphs of bounded MSC is $\mathcal{O}(p \cdot k)$

Lock (Causality) Graphs

T_1	T_2
a0: lock(l_3);	b0: lock(l_1);
a1: lock(l_1);	b1: notify(c);
a2: wait$_{pre}$(c)	b2: unlock(l_1);
a3: unlock(l_1)	b3: lock(l_1);
a4: lock(l_1)	b4: lock(l_3);
a5: wait$_{post}$(c);	b5: unlock(l_1);
a6: lock(l_2);	b6: lock(l_2);
a7: unlock(l_3);	b7: unlock(l_2);
a8: unlock(l_1);	b8: unlock(l_3);
a9: $sh = sh + 1$;	b9: $sh = sh + 2$;
a10: unlock(l_2);	b10: ...

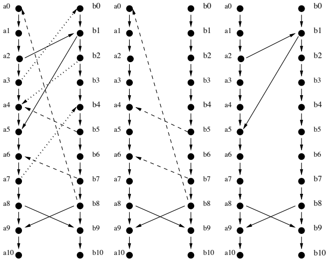


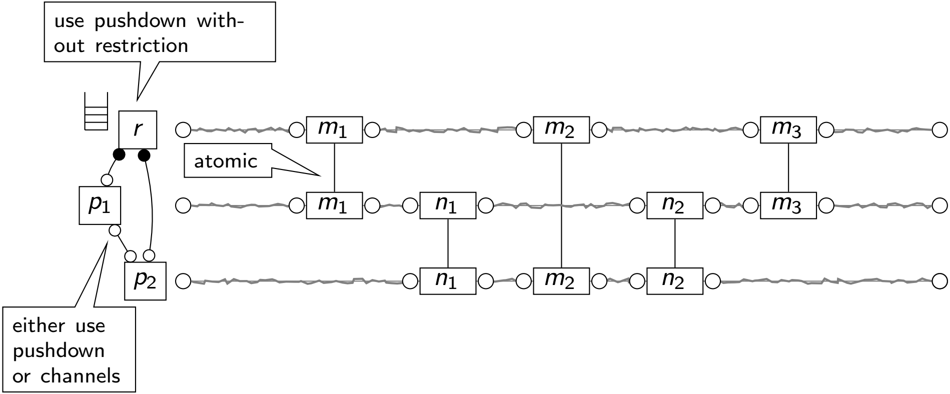
Fig. 1. An Example Universal Causality Graph

figure taken from their CAV'10 paper

- introduced in [Kahlon/Wang '10]
- encode in non-terminals who holds and who requests lock
- add additional back-causality edges (from last holder to new)
- possible in $\mathcal{O}(p \cdot l)$ for p processes and l locks

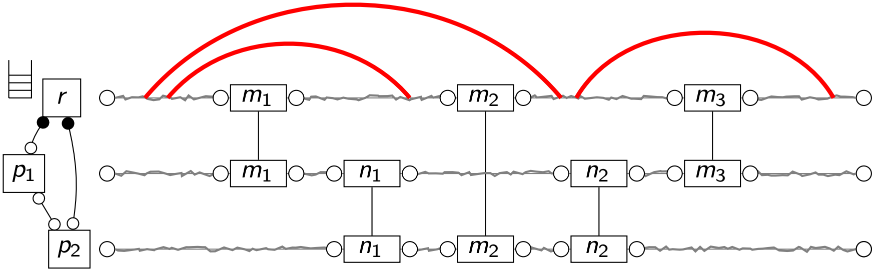
Well-queueing Eager Non-confluent RQCP

➪ decidability depends on the following restrictions [HLMS '10]



Well-queueing Eager Non-confluent RQCP

➪ take a closer look at interplay of pushdowns and sync



Well-queueing Eager Non-confluent RQCP

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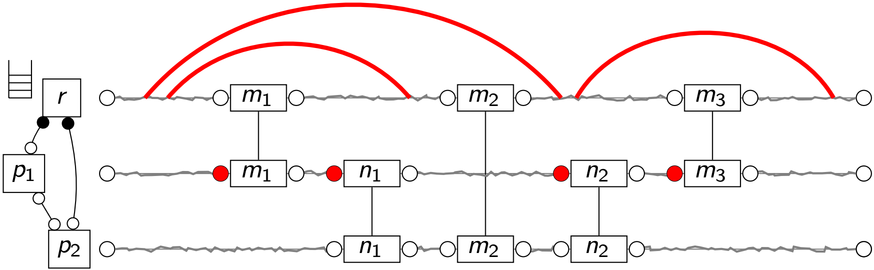

Well-queueing Eager Non-confluent RQCP

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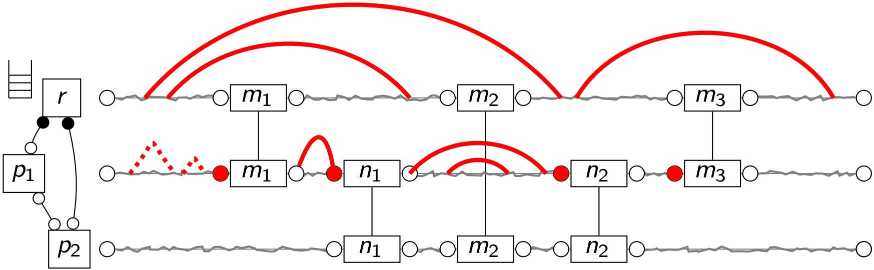

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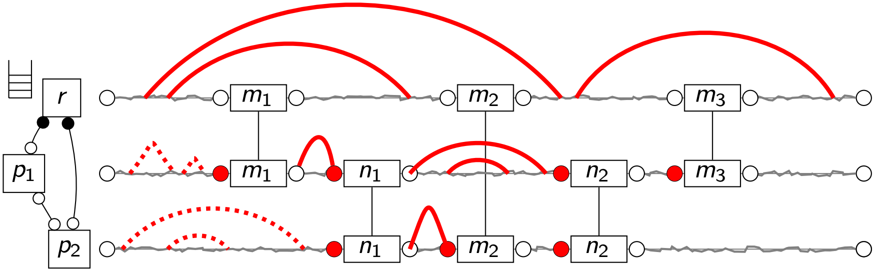
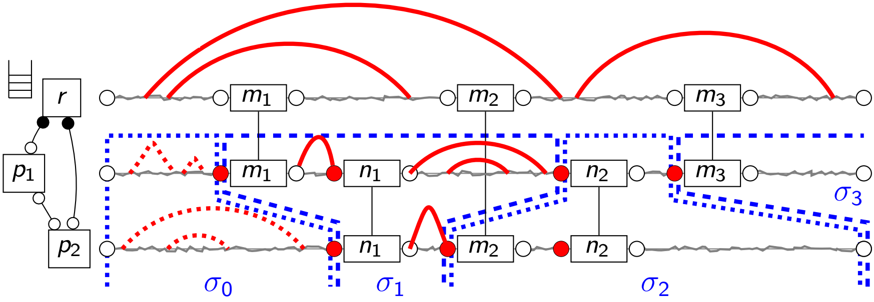


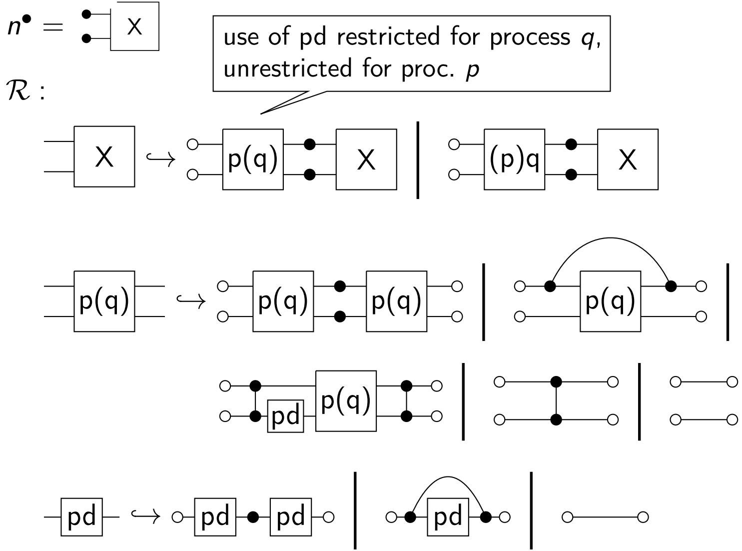
Well-queueing Eager Non-confluent RQCP

➪ "cut" the run to simulate it on one pushdown

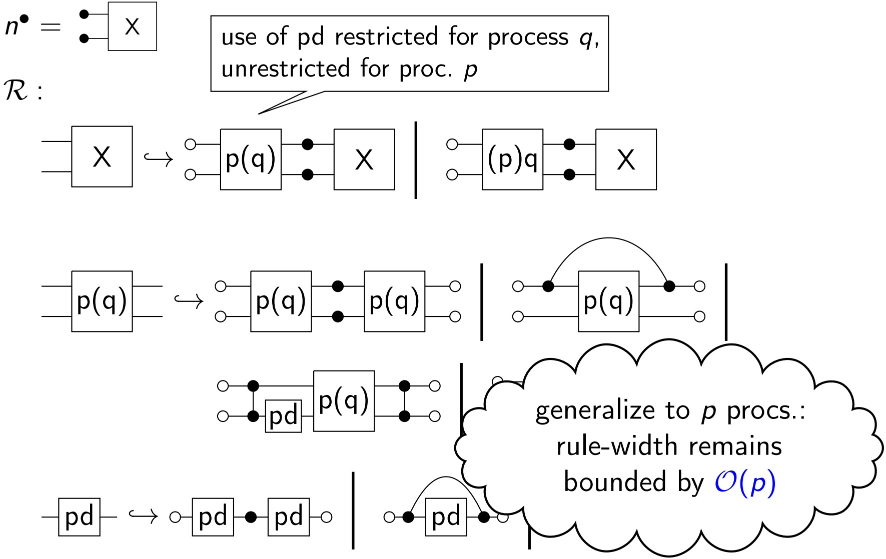


➪ more general class of 1-pd-simulateable systems in [Atig '10]

. . . a HRG for the Two Process Setting



. . . a HRG for the Two Process Setting



Summary/Outlook

- ➪ retrace known results in a simple "visual" way (*intuitions!*)
- ➪ give several new results of MSO-decidable QCP classes
- ➪ framework to "pre-test" restrictions for QCP wrt. decidability

- ➪ more detailed complexity results missing in approach
- ➪ what about "beneath the stars": μ-calculus, LTL, simple reachability, and their relation to graph grammars ?
- ➪ from MSO on *one* run to *branching* runs ?
- ➪ can we extend these ideas to a dynamic setting ?
- ➪ catalogue of anti-patterns ?

- ➪ please feel free to append your ideas/remarks to this list. . .