Expand, Enlarge, and Check
New algorithms for the coverability problem of WSTS*

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Abstract. In this paper, we present a general algorithmic schema called "Expand, Enlarge and Check" from which new efficient algorithms for the coverability problem of WSTS can be constructed. We show here that our schema allows us to define forward algorithms that decide the coverability problem for several classes of systems for which the Karp and Miller procedure cannot be generalized, and for which no complete forward algorithms were known. Our results have important applications for the verification of parameterized systems and communication protocols.

1 Introduction

Model-checking is nowadays widely accepted as a powerful technique for the automatic verification of reactive systems that have natural finite state abstractions. However, many reactive systems are only naturally modelled as infinite-state systems. Consequently, a large (and successful) research effort has recently focused on the application of model-checking techniques to infinite-state models such as FIFO channel systems [2], Petri nets [15], broadcast protocols [7], etc.

One of the positive results is the decidability of the *coverability problem* for *well-structured transition systems* (WSTS for short). WSTS enjoy an infinite set of states that is well-quasi ordered by $\leq$ and their transition relation is monotonic w.r.t $\leq$. Examples of such systems are Petri nets and their monotonic extensions [5, 15], broadcast protocols [8], lossy channel systems [2]. The *coverability problem* asks, given two states c_1 and c_2, whether there is $c_3 \geq c_2$ (c_3 covers c_2) that is reachable from c_1.

A general algorithm (i.e. a procedure that always terminates) is known to solve the coverability problem for WSTS [1, 10]. It symbolically manipulates upward-closed sets of states, obtained by unrolling the transition relation in a *backward* fashion. Unfortunately, backward search is seldom efficient in practise [12], and the only complete forward approach known so far is the Karp-Miller algorithm that can only be applied to a small subclass of WSTS: Petri nets.

The Karp and Miller procedure computes, through a combination of a forward exploration strategy and a simple acceleration technique, the so-called *covering set* of the net, which is known to be well-suited to decide the coverability problem. After several attempts to generalize this procedure to WSTS

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(which have all produced incomplete approaches [8, 9]), it has been shown in [6] that Petri nets form the sole class (among the examples cited above) for which the covering set is constructible in general. However, this set always exists and is usually finitely representable. Our *main contribution* is to make the best of this fact and devise a forward technique that is complete to decide the coverability problem for a large class of WSTS. This class includes, among others, all the monotonic extensions of Petri nets defined in the literature, as well as lossy channel systems.

We present a new schema of algorithm: "Expand, Enlarge and Check" that works by iteratively constructing more and more precise abstractions of the system. These abstractions (made up of reachable states and limit elements) are guaranteed to become precise enough to decide the coverability problem after a finite number of steps. We show how to apply the schema on two classes of WSTS of practical interest: monotonic extensions of Petri nets (that are useful to model parameterized systems [11, 15]) and lossy channels systems (that are useful to model communication protocols [2]).

Due to lack of space, most of the proofs have been omitted. A complete version of the paper can be found at:
http://www.ulb.ac.be/di/ssd/cfv/TechReps/TechRep_CFV_2004_25.pdf

2 Preliminaries

In this section, we recall some fundamental results about *well-quasi orderings* and *well-structured transition systems* (the systems we analyze here). We show how to *finitely* represent upward- and downward-closed sets of states (which will allow us to devise *symbolic* algorithms), and discuss And-Or graphs (useful to represent abstractions of systems).

Well quasi-orderings and adequate domains of limits A *well quasi ordering* $\leq$ on the elements of a set C (wqo for short) is a *reflexive* and *transitive* relation such that for any infinite sequence $c_0 c_1 \ldots c_n \ldots$ of elements in C, there exist two indices i and j, such that $i < j$ and $c_i \leq c_j$. In the following, we note $c_i < c_j$ if $c_i \leq c_j$ but $c_j \not\leq c_i$.

Let $\langle C, \leq \rangle$ be a well-quasi ordered set. A $\leq$-*upward closed set* $U \subseteq C$ is such that for any $c \in U$, for any $c' \in C$ such that $c \leq c'$, $c' \in U$. A $\leq$-*downward closed set* $D \subseteq C$ is such that for any $c \in D$, for any $c' \in C$ such that $c' \leq c$, $c' \in D$. It is well-known that any $\leq$-upward closed set $U \subseteq C$ is uniquely determined by its finite sets of minimal elements. Formally, the set of $\leq$-*minimal* elements $\mathsf{Min}(U)$ of a set $U \subseteq C$ is a minimal set such that $\mathsf{Min}(U) \subseteq U$ and $\forall s' \in U : \exists s \in \mathsf{Min}(U) : s \leq s'$. The next proposition is a consequence of wqo:

Proposition 1. *Let $\langle C, \leq \rangle$ be a wqo set and $U \subseteq C$ be an $\leq$-upward closed set, then: $\mathsf{Min}(U)$ is finite and $U = \{c \mid \exists c' \in \mathsf{Min}(U) : c' \leq c\}$.*

Thus, any $\leq$-upward closed set can be *effectively represented* by its finite set of minimal elements. To obtain a finite representation of downward-closed sets,

we must use well-chosen limit elements $\ell \notin C$ to represent downward closures of infinite increasing chains of elements. Thus, we introduce the notion of *adequate* domain of limits.

Definition 1. Let $\langle C, \leq \rangle$ be a well-quasi ordered set and L be a set of elements disjoint from C, the tuple $\langle L, \sqsubseteq, \gamma \rangle$ is called an *adequate domain of limits* for $\langle C, \leq \rangle$ if the following conditions are satisfied: ($\mathsf{L_1}$: representation mapping) $\gamma : L \cup C \to 2^C$ associates to each element in $L \cup C$ a $\leq$-downward closed set $D \subseteq C$, furthermore, for any $c \in C$, we impose that $\gamma(c) = \{c' \mid c' \leq c\}$. In the following, γ is extended to sets $\mathcal{S} \subseteq L \cup C$ in the natural way: $\gamma(\mathcal{S}) = \cup_{c \in \mathcal{S}} \gamma(c)$; ($\mathsf{L_2}$: top element) There exists a special element $\top \in L$ such that $\gamma(\top) = C$; ($\mathsf{L_3}$: precision order) The elements of $C \cup L$ are ordered by the complete quasi order $\sqsubseteq$, defined as follows: $d_1 \sqsubseteq d_2$ if and only if $\gamma(d_1) \subseteq \gamma(d_2)$; ($\mathsf{L_4}$: completeness) for any downward closed set $D \subseteq C$, there exists a finite set $D' \subseteq C \cup L$ with $\gamma(D') = D$.

Well-structured transition systems and coverability problem A *transition system* is a tuple $S = \langle C, c_0, \to \rangle$ where C is a (possibly infinite) set of states, $c_0 \in C$ is the initial state, $\to \subseteq C \times C$ is a transition relation. In the following, $c \to c'$ will denote that $\langle c, c' \rangle \in \to$. For any state c, $\mathsf{Post}(c)$ denotes the set of one-step successors of c, i.e. $\mathsf{Post}(c) = \{c' | c \to c'\}$. We require $\mathsf{Post}(c) \neq \emptyset$ for any $c \in C$[1]. This operator is extended to sets of states $C' \subseteq C$ as follows: $\mathsf{Post}(C') = \{c | \exists c' \in C' : c' \to c\}$. A *path* of S is a sequence of states $c_1, c_2, \ldots, c_k$ such that $c_1 \to c_2 \to \cdots \to c_k$. A state c' is reachable from a state c, noted $c \to^* c'$, if we have a path $c_1, c_2, \ldots c_k$ in S with $c_1 = c$ and $c_k = c'$. Given a transition system $S = \langle C, c_0, \to \rangle$, $\mathsf{Reach}(S)$ denotes the set $\{c \in C \mid c_0 \to^* c\}$.

Definition 2. A transition system $S = \langle C, c_0, \to \rangle$ is a *well-structured transition system* for the quasi order $\leq \subseteq C \times C$ if the two following properties hold: ($\mathsf{W_1}$: well-ordering) $\leq$ is a well-quasi ordering and ($\mathsf{W_2}$: monotonicity) for all $c_1, c_2, c_3 \in C$ such that $c_1 \leq c_2$ and $c_1 \to c_3$, there exists $c_4 \in C$ such that $c_3 \leq c_4$ and $c_2 \to c_4$.

From now on, $S = \langle C, c_0, \to, \leq \rangle$ will denote the well-structured transition system $\langle C, c_0, \to \rangle$ for $\leq$. In the sequel, we need to manipulate WSTS and adequate domain of limits. In particular, we need the following effectiveness properties:

Definition 3. A WSTS $S = \langle C, c_0, \to, \leq \rangle$ and an adequate domain of limits $\langle L, \sqsubseteq, \gamma \rangle$ are *effective* if the following conditions are satisfied: ($\mathsf{E_1}$) C and L are recursively enumerable; ($\mathsf{E_2}$) for any $c_1, c_2 \in C$, we can decide whether $c_1 \to c_2$; ($\mathsf{E_3}$) for any two finite subsets $C' \subseteq C$ and $L' \subseteq L$, for any $d \in C' \cup L'$ and any finite subset $D \subseteq C' \cup L'$, we can decide whether $\mathsf{Post}(\gamma(d)) \subseteq \gamma(D)$; ($\mathsf{E_4}$) For any finite subsets $D_1, D_2 \subseteq C \cup L$, we can decide whether $\gamma(D_1) \subseteq \gamma(D_2)$.

Problem 1. The *coverability problem for well-structured transition systems* is defined as follows: "Given a well-structured transition system S and the $\leq$-upward closed set $U \subseteq C$, determine whether $\mathsf{Reach}(S) \cap U \neq \emptyset$?"

[1] Note that this condition is not restrictive since we can always add a transition to a dummy state.

To solve the coverability problem, we use covering sets, defined as follows:

Definition 4. Let $S = \langle C, c_0, \rightarrow, \leq \rangle$ be a WSTS. The *covering set* of S, noted $\mathsf{Cover}(S)$, is the (unique) smallest subset of C which ($\mathsf{CS_1}$) is $\leq$-downward closed and ($\mathsf{CS_2}$) contains $\mathsf{Reach}(S)$.

Property For any WSTS $S = \langle C, c_0, \rightarrow, \leq \rangle$ with an adequate domain of limits $\langle L, \sqsubseteq, \gamma \rangle$ for $\langle C, \leq \rangle$, by property $\mathsf{L_4}$ of Definition 1, there exists a finite subset $\mathsf{CS}(S) \subseteq L \cup C$ such that $\gamma(\mathsf{CS}(S)) = \mathsf{Cover}(S)$. In the following, $\mathsf{CS}(S)$ is called a *coverability set* of the covering set $\mathsf{Cover}(S)$ and finitely represents that set.

Proposition 2. *For any* WSTS $S = \langle C, c_0, \rightarrow, \leq \rangle$*, the covering set of* S *is such that for any* $\leq$*-upward closed set* $U \subseteq C$*:* $\mathsf{Reach}(S) \cap U = \emptyset$ *iff* $\mathsf{Cover}(S) \cap U = \emptyset$*.*

And-Or graph and its avoidability problem An *And-Or graph* is a tuple $G = \langle V_A, V_O, v_i, \Rightarrow \rangle$ where $V = V_A \cup V_O$ is the set of nodes (V_A is the set of "And" nodes and V_O is the set of "Or" nodes), $V_A \cap V_O = \emptyset$, $v_i \in V_O$ is the initial node, and $\Rightarrow \subseteq (V_A \times V_O) \cup (V_O \times V_A)$ is the transition relation such that for any $v \in V_A \cup V_O$, there exists $v' \in V_A \cup V_O$ such that $(v, v') \in \Rightarrow$.

Definition 5. A *compatible unfolding* of an And-Or graph $G = \langle V_A, V_O, v_i, \Rightarrow \rangle$ is an infinite labelled tree $T_G = \langle N, root, B, \Lambda \rangle$ where: (i) N is the set of nodes of T_G, (ii) $root \in N$ is the root of T_G, (iii) $B \subseteq N \times N$ is the transition relation of T_G, (iv) $\Lambda : N \rightarrow V_A \cup V_0$ is the labelling function of the nodes of T_G by nodes of G that respects the three following compatibility conditions (Λ is extended to sets of nodes in the usual way): ($\mathsf{C_1}$)$\Lambda(root) = v_i$; ($\mathsf{C_2}$) for all $n \in N$ such that $\Lambda(n) \in V_A$, we have that (a) for all nodes $v' \in V_O$ such that $\Lambda(n) \Rightarrow v'$, there exists one and only one $n' \in N$ such that $B(n, n')$ and $\Lambda(n') = v'$, and conversely (b) for all nodes $n' \in N$ such that $B(n, n')$, there exists $v' \in V_O$ such that $\Lambda(n) \Rightarrow v'$ and $\Lambda(n') = v'$. ($\mathsf{C_3}$) for all $n \in N$ such that $\Lambda(n) \in V_O$, we have that: there exists one and only one $n' \in N$ such that $B(n, n')$, and $\Lambda(n) \Rightarrow \Lambda(n')$.

Problem 2. The *And-Or Graph Avoidability Problem* is defined as follows: "Given an And-Or graph $G = \langle V_A, V_O, v_i, \Rightarrow \rangle$ and a set $E \subseteq V_A \cup V_O$, does there exist $T = \langle N, root, \Lambda, B \rangle$, a compatible unfolding of G, such that $\Lambda(N) \cap E = \emptyset$?". When the answer is positive, we say that E is *avoidable* in G.

It is well-known that this problem is complete for *PTIME*.

3 A new schema of algorithms

In this section, we introduce our new schema of algorithms to decide the coverability problem for WSTS. We first explain, in subsection 3.1, how to build an *abstraction* of a given WSTS, w.r.t. a given finite set of reachable states $C' \subseteq C$ and a given finite set of limit elements $L' \subseteq L$. These abstractions are *And-Or graphs* whose nodes are annotated by downward-closed sets of states of a WSTS. We show in subsection 3.2 that any unfolding of this And-Or graph is able to

simulate the behaviours of its associated WSTS (Proposition 3). Moreover, if the downward-closed sets that are used to annotate the And-Or graph are *precise enough* (in a sense that we make clear in Theorem 2), then the And-Or graph can be used to decide *negative instances* of the coverability problem. Based on those results, we propose a new algorithmic schema to decide the coverability problem of WSTS. It works by iteratively constructing abstractions of the WSTS which become more and more precise. In parallel, it also explores, in a breadth-first fashion, the set of reachable states of the system (to be able to decide the *positive instances* of the problem). Thus, after a finite number of steps either a concrete trace to a *covering state* will be found, or *precise enough abstraction* will be computed to prove that no covering state can ever be reached.

3.1 The And-Or Graph $\mathsf{Abs}(S, C', L')$

Definition 6. Given a WSTS $S = \langle C, c_0, \rightarrow, \leq \rangle$, an adequate domain of limits $\langle L, \sqsubseteq, \gamma \rangle$ for $\langle C, \leq \rangle$, a finite subset $C' \subseteq C$ with $c_0 \in C'$, and a finite subset $L' \subseteq L$ with $\top \in L'$, the And-Or graph $G = \langle V_A, V_O, v_i, \Rightarrow \rangle$, noted $\mathsf{Abs}(S, C', L')$, is defined as follows: ($\mathsf{A_1}$) $V_O = C' \cup L'$; ($\mathsf{A_2}$) $V_A = \{S \in 2^{L' \cup C'} \setminus \{\emptyset\} \mid \nexists d_1 \neq d_2 \in S : d_1 \sqsubseteq d_2\}$; ($\mathsf{A_3}$) $v_i = c_0$; ($\mathsf{A_{4.1}}$) $(n_1, n_2) \in \Rightarrow$ with $n_1 \in V_A, n_2 \in V_O$ if and only if $n_2 \in n_1$; ($\mathsf{A_{4.2}}$) for any $n_1 \in V_O, n_2 \in V_A : (n_1, n_2) \in \Rightarrow$ if and only if (i) *successor covering:* $\mathsf{Post}(\gamma(n_1)) \subseteq \gamma(n_2)$, (ii) *preciseness:* $\nexists n \in V_A : \mathsf{Post}(\gamma(n_1)) \subseteq \gamma(n) \subset \gamma(n_2)$.

The following lemma states that the And-Or graph can be constructed for any WSTS and adequate domain of limits that are effective.

Lemma 1. *Given a* WSTS $S = \langle C, c_0, \rightarrow, \leq \rangle$ *and an adequate domain of limits* $\langle L, \sqsubseteq, \gamma \rangle$ *for* $\langle C, \leq \rangle$ *that are effective, a finite subset* $C' \subseteq C$ *with* $c_0 \in C'$*, and a finite subset* $L' \subseteq L$ *with* $\top \in L'$*,* $\mathsf{Abs}(S, C', L')$ *is effectively constructible.*

Notice that in $\mathsf{Abs}(S, C', L')$ all the nodes have at least one successor. Indeed, for all $n \in V_A$, since $n \neq \emptyset$ (following point $\mathsf{A_{4.1}}$ and point $\mathsf{A_2}$ of Definition 6), n has at least one successor. Since And-nodes are subsets of limits that may contain the $\top$ element, with $\gamma(\top) = C$ (following point $\mathsf{L_2}$ of Definition 1), we can always approximate for any $n \in V_O$ the (non-empty) set of successors of $\gamma(n)$, hence we are guaranteed to have at least one successor of n (point $\mathsf{A_{4.2}}$ of Definition 6).

Given a WSTS $S = \langle C, c_0, \rightarrow, \leq \rangle$, an associated And-Or graph $\mathsf{Abs}(S, L', C') = \langle V_A, V_O, v_i, \Rightarrow \rangle$, and an $\leq$-upward-closed set of states $U \subseteq C$, we note $\mathsf{Abs}(U)$ the set of nodes $v \in V_A \cup V_O$ such that $\gamma(v) \cap U \neq \emptyset$, that is, the set of nodes whose associated downward-closed set of states intersects with U. It is easy to show that this subset of nodes can be effectively computed for any effective WSTS with adequate domain of limits.

Degenerated case If an And-Or graph is such that any Or-node has exactly one successor, the And-Or graph is said to be *degenerated*. In that case, the avoidability problem is equivalent to the (un)reachability problem in a plain

graph. From the definition of $\mathsf{Abs}(S, C', L')$, we remark that the And-Or graph will be degenerated if for any $d \in C' \cup L'$, there exists a *unique* minimal set $\gamma(D)$ such that $D \in V_A$ and $\mathsf{Succ}(\gamma(d)) \subseteq \gamma(D)$. This motivates the next definition:

Definition 7. Given a WSTS $S = \langle C, c_0, \rightarrow, \leq \rangle$ and an adequate domain of limits $\langle L, \sqsubseteq, \gamma \rangle$ for $\langle C, \leq \rangle$, we say that a pair $\langle C', L' \rangle$, where $C' \subseteq C$ with $c_0 \in C$ and $L' \subseteq L$ with $\top \in L'$, is *perfect* if for any $d \in C' \cup L'$, there exists a unique minimal set $D \subseteq C' \cup L'$ such that (i) $\mathsf{Post}(\gamma(d)) \subseteq \gamma(D)$ and (ii) there is no $D' \subseteq C' \cup L'$ with $\mathsf{Post}(\gamma(d)) \subseteq \gamma(D') \subset \gamma(D)$.

Lemma 2. *Given a* WSTS $S = \langle C, c_0, \rightarrow, \leq \rangle$*, an adequate domain of limits* $\langle L, \sqsubseteq, \gamma \rangle$ *for* $\langle C, \leq \rangle$*, a finite subset* $C' \subseteq C$ *with* $c_0 \in C'$*, and a finite subset* $L' \subseteq L$ *with* $\top \in L'$ *such that* $\langle C', L' \rangle$ *is perfect, then* $\mathsf{Abs}(S, C', L')$ *is a degenerated And-Or graph.*

3.2 Properties of $\mathsf{Abs}(S, C', L')$

In this section, we prove important properties of $\mathsf{Abs}(S, C', L')$. Roughly speaking, we prove now that the abstraction we have defined above is *adequate* for any pair $\langle C', L' \rangle$ such that $c_0 \in C'$ and $\top \in L'$ (Theorem 1) and *complete* (Theorem 2) for some pair $\langle C', L' \rangle$. To establish those results, we first show that $\mathsf{Abs}(S, C', L')$ can simulate for any $\langle C', L' \rangle$ such that $c_0 \in C'$ and $\top \in L'$ its underlying WSTS.

Proposition 3 (Simulation). *Given a* WSTS $S = \langle C, c_0, \rightarrow, \leq \rangle$ *with an adequate domain of limits* $\langle L, \sqsubseteq, \gamma \rangle$ *for* $\langle C, \leq \rangle$*, the following holds for any* $C' \subseteq C$ *with* $c_0 \in C'$ *and* $L' \subseteq L$ *with* $\top \in L'$*: for any path* $c_0 c_1 \dots c_k$ *of* S *and any unfolding* $T = \langle N, root, B, \Lambda \rangle$ *of* $\mathsf{Abs}(S, C', L')$ *there exists a path* $n_0 n_1 \dots n_{2k}$ *of* T *with* $n_0 = root$ *and such that* $c_i \in \gamma(\Lambda(n_{2i}))$ *for* $0 \leq i \leq k$*.*

Since any unfolding of $\mathsf{Abs}(S, C', L')$ can simulate $S = \langle C, c_0, \rightarrow, \leq \rangle$ for any C', L' with $c_0 \in C'$ and $\top \in L'$, for any upward-closed set $U \subseteq C$ we know that if $\mathsf{Abs}(U)$ is avoidable in $\mathsf{Abs}(S, C', L')$ then U does not intersect with $\mathsf{Reach}(S)$. That is formally stated by the next theorem.

Theorem 1 (Adequacy). *Given a* WSTS $S = \langle C, c_0, \rightarrow, \leq \rangle$*, an adequate domain of limits* $\langle L, \sqsubseteq, \gamma \rangle$ *for* $\langle C, \leq \rangle$*, and an upward-closed set* $U \subseteq C$*, the following holds for any* $C' \subseteq C$ *with* $c_0 \in C'$ *and* $L' \subseteq L$ *with* $\top \in L'$*:* if $\mathsf{Abs}(U)$ *is avoidable in* $\mathsf{Abs}(S, C', L')$*,* then $\mathsf{Reach}(S) \cap U = \emptyset$*.*

Finally, we prove the *completeness* of our approach. Intuitively, the next theorem puts forward that, when the pair $\langle C', L' \rangle$ is *precise enough*, $\mathsf{Abs}(S, C', L')$ allows us to decide *negative instances* of the coverability problem.

Theorem 2 (Completeness). *Given a* WSTS $S = \langle C, c_0, \rightarrow, \leq \rangle$*, an adequate domain of limits* $\langle L, \sqsubseteq, \gamma \rangle$ *for* $\langle C, \leq \rangle$ *and an upward closed set* $U \subseteq C$*, the following holds for any* $C' \subseteq C$ *with* $c_0 \in C'$ *and* $L' \subseteq L$ *with* $\top \in L'$ *such that* $\mathsf{CS}(S) \subseteq C' \cup L'$*:* if $\mathsf{Reach}(S) \cap U = \emptyset$ then $\mathsf{Abs}(U)$ *is avoidable in* $\mathsf{Abs}(S, C', L')$*.*

$i := 0$;
while *(*`true`*)* **do**
  "Expand" Compute S_i;
  "Enlarge" Compute L_i;
  "Check" **if** $\exists c_1, \ldots, c_k : c_0 \rightarrow \ldots \rightarrow c_k$ *with* $c_j \in S_i$ *for all* $0 \leq j \leq k$ *and* $c_k \in U$ **then return** "Reachable";
  else if $\mathsf{Abs}(U)$ *is avoidable in* $\mathsf{Abs}(S, S_i, L_i)$ **then return** "Unreachable";

Fig. 1: **Abstract algorithm** Its inputs are an effective representation of a WSTS $S = \langle C, c_0, \rightarrow, \leq \rangle$ with the adequate limit domain $\langle L, \sqsubseteq, \gamma \rangle$ for $\langle C, \leq \rangle$ and a finite representation of the upward-closed set of states $U \subseteq C$.

3.3 The new algorithmic schema

Let $S_0, S_1, \ldots, S_n \ldots$ be an infinite sequence of finite sets of reachable states of S such that (i) $\forall i \geq 0 : S_i \subseteq S_{i+1}$, (ii) $\forall c \in \mathsf{Reach}(S) : \exists i \geq 0 : c \in S_i$, and (iii) $c_0 \in S_0$. Let $L_0, L_1, \ldots, L_n, \ldots$ be a infinite sequence of finite sets of limits such that (i) $\forall i \geq 0 : L_i \subseteq L_{i+1}$, (ii) $\forall \ell \in L : \exists i \geq 0 : \ell \in L_i$ and (iii) $\top \in L_0$. A schema of algorithm is given at Figure 1 and its correctness is stated in Theorem 3.

Theorem 3. *For any* WSTS S *with adequate domain of limits* $\langle L, \sqsubseteq, \gamma \rangle$ *that are effective, for any upward-closed set* U *represented by* $\mathsf{Min}(U)$*, Algorithm at Fig. 1 terminates after a finite amount of time and returns "Reachable" if* $\mathsf{Reach}(S) \cap U \neq \emptyset$*, "Unreachable" otherwise.*

Proof. (Sketch) If $\mathsf{Reach}(S) \cap U \neq \emptyset$, we have from Theorem 1 that $\mathsf{Abs}(U)$ is not avoidable in $\mathsf{Abs}(S, S_i, L_i)$ for all $i \geq 0$. Moreover, since for all $c \in \mathsf{Reach}(S)$ there exists j such that $c \in S_{j'}$ for all $j' \geq j$, there exists $i \geq 0$ such that we have $c_0 \rightarrow \ldots \rightarrow c_k$ with $c_j \in S_i$ for all j such that $0 \leq j \leq k$ and $c_k \in U$. We conclude that the algorithm at Fig. 1 returns "Reachable" if $\mathsf{Reach}(S) \cap U \neq \emptyset$.

If $\mathsf{Reach}(S) \cap U = \emptyset$, we know that there exists $i \geq 0$ and a finite coverability set $\mathsf{CS}(S)$ such that $\mathsf{CS}(S) \subseteq S_i \cup L_i$. Hence, from Theorem 2 we have that $\mathsf{Abs}(U)$ is avoidable in $\mathsf{Abs}(S, S_i, L_i)$ and we conclude that the algorithm at Fig. 1 returns "Unreachable" if $\mathsf{Reach}(S) \cap U = \emptyset$. □

Remark 1. Note that Theorem 3, that states the adequation and completeness of our algorithmic schema for the coverability problem of effective WSTS, is not in contradiction with the result of [6] which establishes that there does not exist a procedure that always terminates and returns a coverability set for a large class of WSTS, including ours. Indeed, to establish the correctness of our algorithm, we only need to ensure that a coverability set will be included at some point in the sequence of S_i's and L_i's. Nevertheless, given a pair $\langle S_i, L_i \rangle$, it is not possible to establish algorithmically that this pair contains a coverability set. Also, given a particular upward-closed set U, our algorithm may terminate before reaching a pair $\langle S_i, L_i \rangle$ that contains a coverability set, because the set U is reachable or because the abstraction constructed from a pair $\langle S_j, L_j \rangle$, with $j < i$, is sufficiently precise to prove that U is not reachable.

Remark 2. Note that the constraints on the sequence of L_i's computed by the algorithm of Fig. 1 may be relaxed. Indeed, those constraints ensure that the algorithm eventually considers a set of limits which allows to construct a graph that is precise enough to decide negative instances of the coverability problem. However, following Theorem 2, it is sufficient to ensure that there exists $i \geq 0$ such that $S_i \cup L_i$ contains a coverability set. Hence, only the limits of a coverability set must appear in the sequence of L_i's.

4 Application to Self-modifying Petri nets

Let us show how to apply the approach proposed in the previous section to solve the coverability problem for a large subclass of *Self-modifying Petri nets* [14] (SMPN). SMPN are a general extension of Petri nets that includes almost all the monotonic extensions of Petri nets defined in the literature and for which, so far, there was no complete forward procedure.

4.1 Self-modifying Petri nets

A *Self-Modifying Petri net* [14], SMPN for short, is a tuple $\langle P, T, D^-, D^+, \mathbf{m}_0 \rangle$. $P = \{p_1, \ldots, p_{k_P}\}$ is a finite (non-empty) set of places. A *marking* is a function $\mathbf{m} : P \to \mathbb{N}$ that assigns a natural value to each place. In the following, markings are also seen as tuples in $\mathbb{N}^{k_P}$ where the ith dimension is the value assigned to place p_i. $T = \{t_1, \ldots, t_{k_T}\}$ is a finite (non-empty) set of transitions. For any $1 \leq i \leq k_T$ and any $1 \leq j \leq k_P$, $D^-_{ij} : \mathbb{N}^{k_P} \to \mathbb{N}$ and $D^+_{ij} : \mathbb{N}^{k_P} \to \mathbb{N}$ describe respectively the input and output effect of transition t_i on place p_j. Namely, D^-_{ij} and D^+_{ij} are functions of the marking $\mathbf{m}$ restricted to the form $\alpha + \sum_{k=1..k_P} \beta_k \cdot \mathbf{m}(p_k)$ where $\alpha \in \mathbb{N}$ and $\beta_k \in \mathbb{N}$ for all $1 \leq k \leq k_P$. $\mathbf{m}_0$ is the initial marking of the SMPN.

We define the quasi order $\preccurlyeq \subseteq \mathbb{N}^{k_P} \times \mathbb{N}^{k_P}$ on markings such that $\langle m_1, \ldots, m_{k_P} \rangle \preccurlyeq \langle m'_1, \ldots, m'_{k_P} \rangle$ if $m_i \leq m'_i$ for all $1 \leq i \leq k_P$. It is well-known that $\preccurlyeq$ is a wqo.

A transition t_i is firable from a marking $\mathbf{m}$ if $\mathbf{m}(p_j) \geq D^-_{ij}(\mathbf{m})$ for all $p_j \in P$. Firing t_i from $\mathbf{m}$ leads to a marking $\mathbf{m'} \in \mathbb{N}^{k_P}$, noted $\mathbf{m} \to_{t_i} \mathbf{m'}$, such that, for any $p_j \in P : \mathbf{m'}(p_j) = \mathbf{m}(p_j) + D^+_{ij}(\mathbf{m}) - D^-_{ij}(\mathbf{m})$. Given a set S of markings and a transition t_i, $\mathsf{Post}(S, t_i) = \{\mathbf{m'} \mid \exists \mathbf{m} \in S : \mathbf{m} \to_{t_i} \mathbf{m'}\}$.

A SMPN $\mathcal{P}$ defines a transition system $\mathcal{T}_\mathcal{P} = \langle \mathbb{N}^{k_P}, \mathbf{m}_0, \to \rangle$ where $\to \subseteq \mathbb{N}^{k_P} \times \mathbb{N}^{k_P}$ is a transition relation and is such that we have $\langle \mathbf{m}, \mathbf{m'} \rangle \in \to$, noted $\mathbf{m} \to \mathbf{m'}$, if and only if there exists $t_i \in T$ such that t_i is firable from $\mathbf{m}$ and $\mathbf{m} \to_{t_i} \mathbf{m'}$.

A SMPN $\mathcal{P}$ is $\preccurlyeq$-*monotonic* when the underlying transition system $\mathcal{T}_\mathcal{P}$ satisfies the monotonicity property for $\preccurlyeq$. A SMPN $\mathcal{P}$ is *strongly monotonic* when for every transition t_i and markings $\mathbf{m}_1, \mathbf{m}_2$ and $\mathbf{m}_3$, the following holds: if $\mathbf{m}_1 \to_{t_i} \mathbf{m}_3$ and $\mathbf{m}_1 \preccurlyeq \mathbf{m}_2$, there exists $\mathbf{m}_4$ such that $\mathbf{m}_2 \to_{t_i} \mathbf{m}_4$ and $\mathbf{m}_3 \preccurlyeq \mathbf{m}_4$. Obviously, all the strongly monotonic SMPN are $\preccurlyeq$-monotonic.

We say that a transition t is *unfirable*, whenever there exists no marking $\mathbf{m}$ such that t is enabled in $\mathbf{m}$. In the following, we assume that the SMPN's we

consider do not contain unfirable transitions. The following lemma defines the syntactical subclass of SMPN's that are strongly monotonic.

Lemma 3. *Given a* SMPN $\mathcal{P} = \langle P, T, D^-, D^+, \mathbf{m}_0 \rangle$ *without unfirable transitions,* $\mathcal{P}$ *is strongly monotonic if and only if for all* $t_i \in T, p_j \in P : D^-_{ij} = \alpha$ *with* $\alpha \in \mathbb{N}$ *or* $D^-_{ij} = \mathbf{m}(p_j)$.

Although strongly monotonic SMPN is a sub-class of SMPN, it remains a general class of monotonic systems. Indeed, almost all the monotonic extensions of Petri nets studied in the literature are syntactical sub-classes of strongly monotonic SMPN, i.e. sub-classes defined by imposing constraints on the linear expressions defining the effect of transitions. Examples of such extensions are Petri nets with transfers [5], with reset [3] and Post self-modifying Petri nets [14]. On the other hand, the other monotonic extensions of Petri nets are not syntactical sub-classes of strongly monotonic SMPN, but we can construct (in polynomial time) a strongly monotonic SMPN with the same set of places that is equivalent to the original net with respect to the coverability problem. Examples of such extensions are Petri nets with non-blocking arcs [13] and Lossy Petri nets [4]. So the algorithm that we propose in the next section is a forward algorithm that decides the coverability problem for all monotonic extensions of Petri nets proposed in the literature.

In the following, we define the adequate domain of limits we consider, state its effectiveness and show how to construct the sequences of S_i's and L_i's. Finally, we show that we always obtain degenerated And-Or graph.

4.2 A forward algorithm to decide the coverability problem for strongly monotonic SMPN

Domain of Limits We will consider the domain of limits $\langle \mathcal{L}, \preccurlyeq_e, \gamma(.) \rangle$ where $\mathcal{L} = (\mathbb{N} \cup \{+\infty\})^k \setminus \mathbb{N}^k$, $\preccurlyeq_e \subseteq (\mathbb{N} \cup \{+\infty\})^k \times (\mathbb{N} \cup \{+\infty\})^k$ is such that $\langle m_1, \ldots, m_k \rangle \preccurlyeq_e \langle m'_1, \ldots, m'_k \rangle$ if and only if $\forall 1 \leq i \leq k : m_i \leq m'_i$ where $c < +\infty$ for all $c \in \mathbb{N}$ ($\leq$ is the natural order over $\mathbb{N} \cup \{+\infty\}$). $\gamma(.)$ is defined as: $\gamma(\mathbf{m}) = \{\mathbf{m'} \in \mathbb{N}^k \mid \mathbf{m'} \preccurlyeq_e \mathbf{m}\}$. In the following, tuples in $\mathcal{L}$ are called extended markings. It is well-known, see for instance [15], that the following lemma holds.

Lemma 4. $\langle \mathcal{L}, \preccurlyeq_e, \gamma(.) \rangle$ *is an adequate domain of limits for* $\langle \mathbb{N}^k, \preccurlyeq \rangle$.

Notice that in this case the $\top$ element such that $\gamma(\top) = \mathbb{N}^k$ is the marking that assigns $+\infty$ to all the places.

Given a strongly monotonic SMPN $\mathcal{P}$, we extend the underlying transition relation from markings to extended markings by assuming that $+\infty + +\infty = +\infty$, $+\infty \cdot c = +\infty$ for all $c \in \mathbb{N} \setminus \{0\}$, $0 \cdot +\infty = 0$, $+\infty + c = +\infty$ for all $c \in \mathbb{Z}$.

Since our algorithm requires the WSTS and its associated domain of limits to be effective (Definition 3), we state the following lemma :

Lemma 5. *Any strongly monotonic* SMPN $\mathcal{P}$ *with the adequate domain of limits* $\langle \mathcal{L}, \preccurlyeq_e, \gamma(.) \rangle$ *are effective.*

$i \leftarrow 1$;
while *(***true***)* **do**
  if $\exists \mathbf{m} \in \mathsf{Reach}_{\mathsf{exact}}(\mathsf{Abs}(\mathcal{P}, i)), \mathbf{m}' \in G_U : \mathbf{m} \preccurlyeq \mathbf{m}'$ **then return** *Reachable*;
  else
    if $\nexists \mathbf{m} \in \mathsf{Reach}(\mathsf{Abs}(\mathcal{P}, i)), \mathbf{m}' \in G_U : \mathbf{m} \preccurlyeq_e \mathbf{m}'$ **then return** *Unreachable*;
    else $i \leftarrow i + 1$;

Fig. 2: **A forward algorithm for** SMPN Its inputs are $\mathcal{P}$, a strongly monotonic SMPN and G_U, the set of minimal elements of the $\preccurlyeq$-upward closed set U.

The following definition explains how we construct the S_i's and L_i's. Following Definition 6, this is sufficient to define the And-Or graphs built by our verification algorithm.

Definition 8. *The sequences of S_i's and L_i's are defined as follows:* (D_1) $S_i = \{0, \ldots, i\}^k \cup \{\mathbf{m}_0\}$*, i.e. S_i is the set of markings where each place is bounded by i (plus the initial marking);* (D_2) $L_i = \{\mathbf{m} \in \{0, \ldots i, +\infty\}^k \mid \mathbf{m} \notin \mathbb{N}^k\}$.

It is easy to see that the S_i's and L_i's are finite sets and (i) for all $i \geq 0$: $S_i \subset S_{i+1}$ and $L_i \subset L_{i+1}$, (ii) for any $\mathbf{m} \in \mathbb{N}^k$, there exists $i \in \mathbb{N}$ such that for all $j \geq i : \mathbf{m} \in S_j$, (iii) for any $\mathbf{m} \in \mathcal{L}$, there exists $i \in \mathbb{N}$ such that for all $j \geq i : \mathbf{m} \in L_j$, and (iv) $\mathbf{m}_0 \in S_0$ and $\top \in L_0$.

Degenerated And-Or graph Let us show that in the present case, one obtains a *degenerated* And-Or graph. For this purpose, we prove, following Lemma 2, that the pairs $\langle S_i, L_i \rangle$ are *perfect* pairs.

Lemma 6. *Given a* SMPN $\mathcal{P} = \langle P, T, D^-, D^+ \rangle$ *with the adequate domain of limits $\langle \mathcal{L}, \preccurlyeq_e, \gamma(.) \rangle$ any pair $\langle S_i, L_i \rangle$, with $S_i \subseteq \mathbb{N}^{k_P}$ and $L_i \subseteq \mathcal{L}$ constructed following Definition 8, is a perfect pair.*

Corollary 1. *Given a strongly monotonic* SMPN *net $\mathcal{P}$ with the adequate domain of limits $\langle \mathcal{L}, \preccurlyeq_e, \gamma(.) \rangle$ and the sets $S_i \subseteq \mathbb{N}^{k_P}$ and $L_i \subseteq \mathcal{L}$ constructed following Definition 8,* $\mathsf{Abs}(\mathcal{P}, S_i, L_i)$ *is a degenerated And-Or graph.*

Algorithm for the coverability problem Let $\mathsf{Abs}(\mathcal{P}, i)$ be the graph (degenerated And-Or graph) $\mathsf{Abs}(\mathcal{P}, S_i, L_i)$ constructed from $\mathcal{P}$, S_i and L_i. We note $\Rightarrow$ its transition relation. We define $\mathsf{Reach}_{\mathsf{exact}}(\mathsf{Abs}(\mathcal{P}, i))$ as the set $\{\mathbf{m} \mid \mathbf{m}_0 \Rightarrow \mathbf{m}_1 \Rightarrow \ldots \Rightarrow \mathbf{m}_n$ with $\forall 1 \leq j \leq n : \mathbf{m}_j \in S_i, \mathbf{m}_n = \mathbf{m}\}$ and $\mathsf{Reach}(\mathsf{Abs}(\mathcal{P}, i))$ as the set $\{\mathbf{m} \mid \mathbf{m}_0 \Rightarrow \mathbf{m}_1 \Rightarrow \ldots \Rightarrow \mathbf{m}_n$ with $\forall 1 \leq j \leq n : \mathbf{m}_j \in S_i \cup L_i, \mathbf{m}_n = \mathbf{m}\}$. By applying the schema presented in Section 3 to strongly monotonic self-modifying Petri nets, we obtain the algorithm at Fig. 2. Remark that this algorithm is *incremental*: one can compute $\mathsf{Reach}_{\mathsf{exact}}(\mathsf{Abs}(\mathcal{P}, i+1))$ by extending $\mathsf{Reach}_{\mathsf{exact}}(\mathsf{Abs}(\mathcal{P}, i))$ for all $i \geq 0$. Similarly, one can construct $\mathsf{Reach}(\mathsf{Abs}(\mathcal{P}, i))$ from $\mathsf{Reach}_{\mathsf{exact}}(\mathsf{Abs}(\mathcal{P}, i))$.

Theorem 4. *For any strongly monotonic* SMPN*, the algorithm of Fig. 2 returns "Reachable" if* $\mathsf{Reach}(\mathcal{C}) \cap U \neq \emptyset$*, "Unreachable" otherwise.*

5 Application to Lossy Channel Systems

To show the generality of our new approach, we apply our schema of algorithm to *lossy channel systems*, which are systems made up of automata extended with FIFO channels that may lose messages. We recall the model, define an adequate domain of limits and show how to construct the sets S_i's and L_i's.

A *Lossy Channel System*, LCS for short, is a tuple $\mathcal{C} = \langle Q, q_i, F, \Sigma, T\rangle$ where Q is a finite set of locations, $q_i \in Q$ is the initial location, F is a finite set of channels, Σ is a finite alphabet, $T \subseteq Q \times Op \times Q$ where $Op : F \mapsto \bigcup_{a\in\Sigma}\{?a, !a\} \cup \{\text{nop}\}$. A *state* is a pair $\langle q, W\rangle$ where $q \in Q$, $W : F \mapsto \Sigma^*$. In the following, $\mathcal{S}_\mathcal{C}$ will denote the 3B set of states of the LCS $\mathcal{C}$. We define the order $\precsim$ on states in $\mathcal{S}_\mathcal{C}$ such that for any $s = \langle q, W\rangle, s' = \langle q', W'\rangle : s \precsim s'$ if and only if $q = q'$ and $W(c)$ is a (not necessarily contiguous) subword of $W'(c)$ for all $c \in F$, i.e $W(c)$ is obtained from $W'(c)$ by deleting characters. It is well-known that $\precsim$ is a well-quasi order (see for instance [1]). A LCS $\langle Q, q_i, F, \Sigma, T\rangle$ defines a transition system $\langle \mathcal{S}_\mathcal{C}, s_0, \rightarrow\rangle$ where (i) $s_0 = \langle q_i, W_i\rangle$ with $W_i(c) = \varepsilon$ for each $c \in F$ and (ii) $(\langle q, W\rangle, \langle q', W'\rangle) \in \rightarrow$ if and only if there exists $t = \langle q_1, Op, q_2\rangle \in T$ and $\langle q, W''\rangle$ with $W'' \precsim W$ such that $q = q_1$, $q' = q_2$ and for all $c \in F : Op(c) = ?a$ implies $W''(c) = a \cdot W'(c)$. Furthermore, $W'(c) = W''(c) \cdot a$ if $Op(c) = !a$ and $W'(c) = W''(c)$ if $Op(c) = \text{nop}$. In the following, we always consider a LCS $\mathcal{C} = \langle Q, q_i, F, \Sigma, T\rangle$.

Domain of limits Let $L(\Sigma)$ be the set of downward closed regular expressions (dc-re) $\{(a_1 + \ldots + a_n)^* \mid \forall 1 \leq i \leq n : a_i \in \Sigma, \forall a_i, a_j : i \neq j$ implies that $a_i \neq a_j\} \cup \{(a + \varepsilon) \mid a \in \Sigma\} \cup \{\varepsilon\}$. A simple regular expression (sre) is either a dc-re or an expression $a_1 \cdot \ldots \cdot a_n$ where $\forall 1 \leq i \leq n : a_i$ is a dc-re. The size of a sre is the number of dc-re that compose it. The set of limits is $\mathcal{L}(\Sigma, Q) = \{\langle q, E\rangle \mid q \in Q, E : F \mapsto L(\Sigma)^*$ assigns a sre to each channel[2]$\} \cup \{\top\}$. For $\langle q, E\rangle \in \mathcal{L}(\Sigma, Q) \setminus \{\varepsilon\}$: $[\![\langle q, E\rangle]\!]$ denotes the set of pairs $\langle q, W\rangle \in \mathcal{S}_\mathcal{C}$ such that $W(c)$ is a word in the language generated by the regular expression $E(c)$ for all $c \in F$. We define the function $\gamma : \mathcal{S}_\mathcal{C} \cup \mathcal{L}(\Sigma, Q) \rightarrow 2^{\mathcal{S}_\mathcal{C}}$ such that (i) for all $\langle q, W\rangle \in \mathcal{S}_\mathcal{C} : \gamma(\langle q, W\rangle) = \{\langle q, W'\rangle \mid \langle q, W'\rangle \precsim \langle q, W\rangle\}$, (ii) $\gamma(\top) = \{\langle q, W\rangle \mid q \in Q, W(c) \in \Sigma^*$ for all $c \in F\}$ and (iii) for all $\langle q, E\rangle \in \mathcal{L}(\Sigma, Q) \setminus \{\top\} : \gamma(\langle q, E\rangle) = [\![\langle q, E\rangle]\!]$. We define $\overline{\sqsubseteq} : (\mathcal{S}_\mathcal{C} \cup \mathcal{L}(\Sigma, Q)) \times (\mathcal{S}_\mathcal{C} \cup \mathcal{L}(\Sigma, Q))$ as follows : $c_1 \overline{\sqsubseteq} c_2$ if and only if $\gamma(c_1) \subseteq \gamma(c_2)$.

It is easy to see that $(\mathcal{L}(\Sigma, Q), \overline{\sqsubseteq}, \gamma)$ is an adequate domain of limits for $(\mathcal{S}_\mathcal{C}, \precsim)$ and that any LCS $\mathcal{C}$ with this domain of limits is effective.

Construction of the S_i's and the L_i's We construct the sequences of the S_i's and L_i's as follows. $S_i = \{\langle q, W\rangle \in \mathcal{S}_\mathcal{C} \mid q \in Q, \forall c \in F : W(c) = \varepsilon$ or $W(c) = a_1 \cdot \ldots \cdot a_n$ with $n \leq i$ and $\forall 1 \leq j \leq n : a_j \in \Sigma\}$, i.e. S_i is the set of states where the contents of the channels are words of size at most i. Similarly, $L_i = \{\langle q, E\rangle \in \mathcal{L}(\Sigma, Q) \mid \forall c \in F : E(c) = \varepsilon$ or $E(c) = e_1 \cdot \ldots \cdot e_n$ with $n \leq i$ and $\forall 1 \leq$

[2] We also require that E does not assign ε to all the channels because we require in Definition 1 that the set of limits be disjoint from $\mathcal{S}_\mathcal{C}$.

$j \leq n : e_j \in L(\Sigma)\} \cup \{\top\}$, i.e. L_i is the set of limits that assign sre of size at most i to the channels (plus the $\top$ element).

It is not difficult to see that the sequences of S_i's and L_i's satisfy the hypothesis of the algorithm of Fig. 1.

6 Conclusion

In this paper, we have defined a new approach to solve the coverability problem of WSTS, which we call "Expand, Enlarge and Check". When applied to a large class of monotonic counter systems (the strong monotonic Self-modifying Petri nets), our approach produces an algorithm that uses forward analysis to decide the coverability problem. Up to now, such a forward approach was known only for Petri nets (the Karp and Miller algorithm), a restricted subclass of strong monotonic SMPN. We have demonstrated the generality of our approach by showing how to apply the algorithmic schema to lossy channel systems.

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