Synthesis of Optimal Strategies for Priced Timed Games

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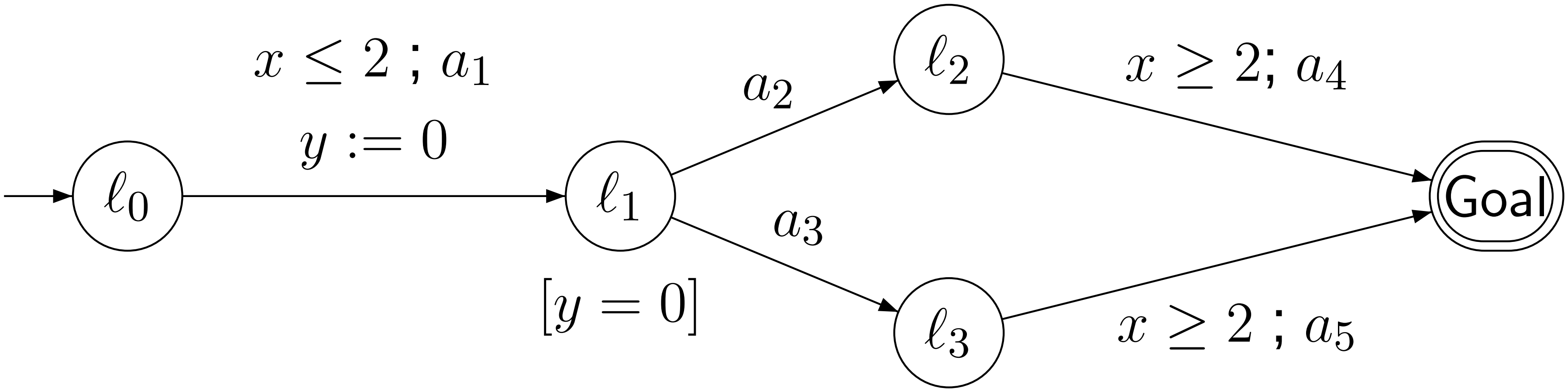
Contents

1. Context & Related Work
2. Priced Timed Game Automata
3. Computing The Optimal Cost
4. Computing Optimal Strategies
5. Implementation using HyTech

Context

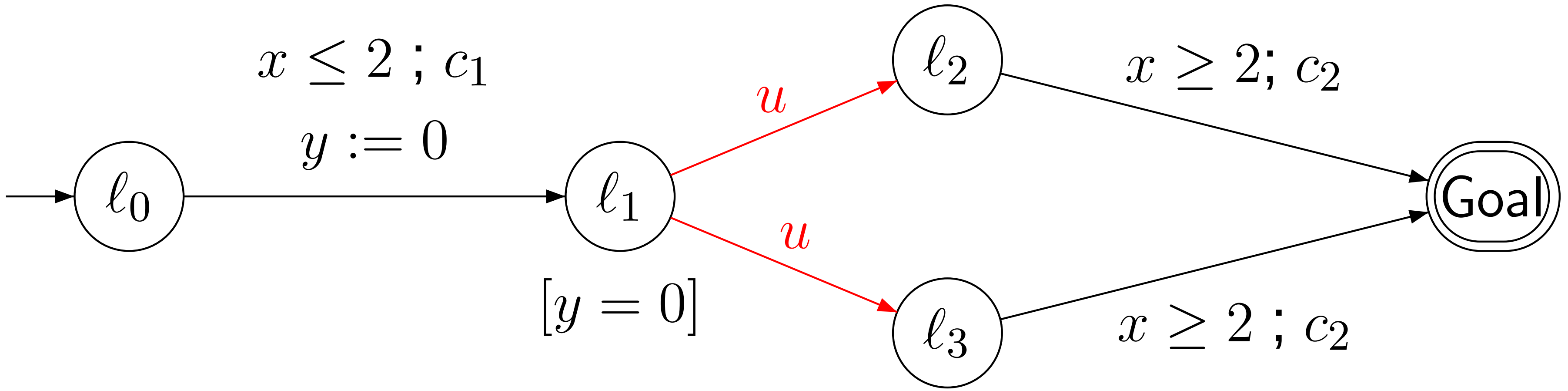
Timed Automata



- Timed Automata + Reachability [AD94]

Context

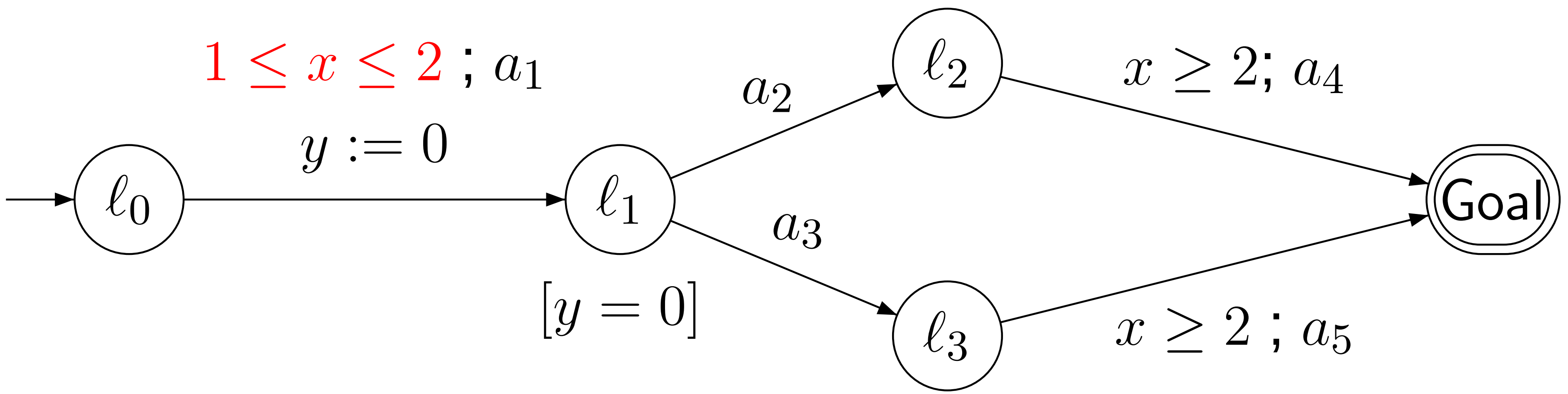
Timed Game Automata



- Timed Automata + Reachability [AD94]
- Timed Game Automata: Control [MPS95, AMPS98]

Context

As soon As Possible in Timed Automata



- Timed Automata + Reachability [AD94]
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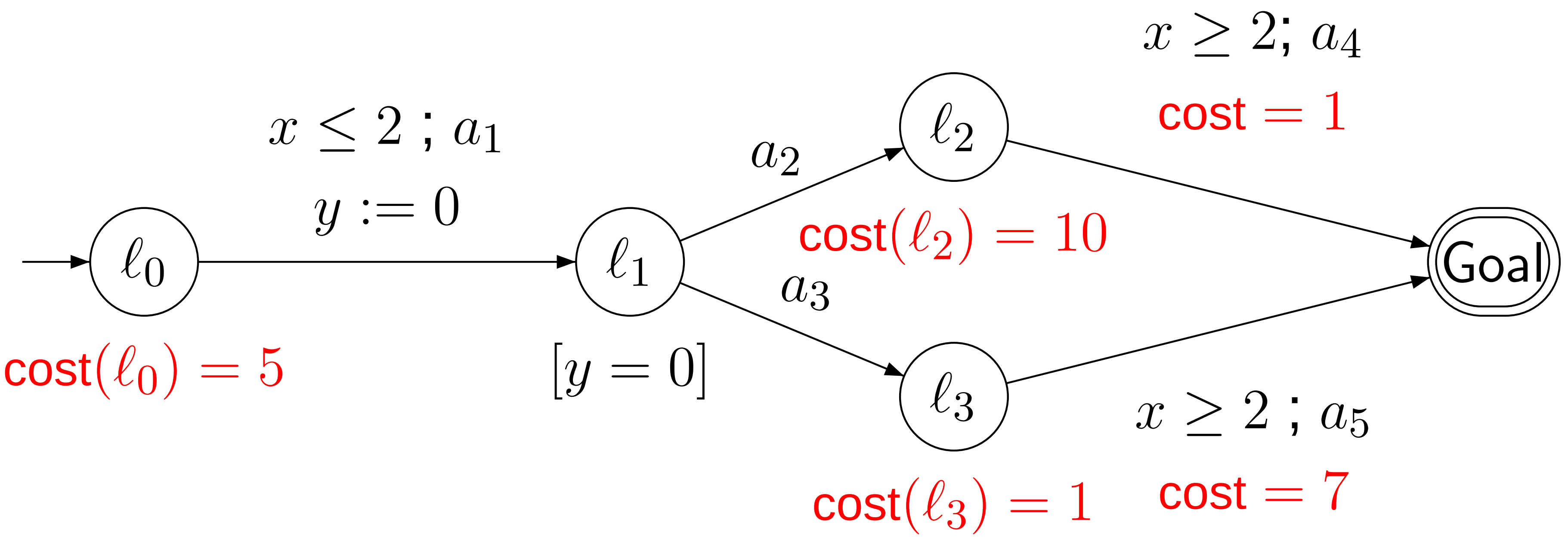
- Time Optimal Reachability [AM99]

Context

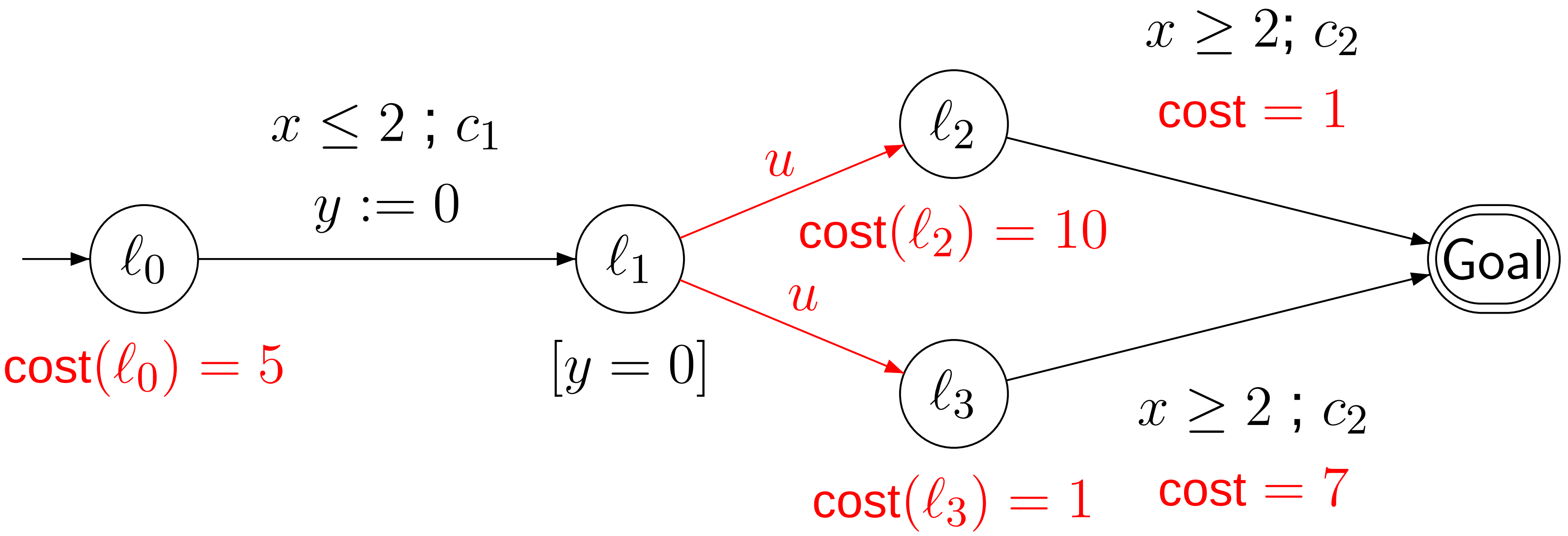
Reachability in Priced Timed Automata



- Timed Automata + Reachability [AD94]
- Timed Game Automata: Control [MPS95, AMPS98]
- Time Optimal Reachability [AM99]
- Priced (or Weighted) Timed Automata [LBB+01, ALTP01]

Context

Priced Timed Game Automata



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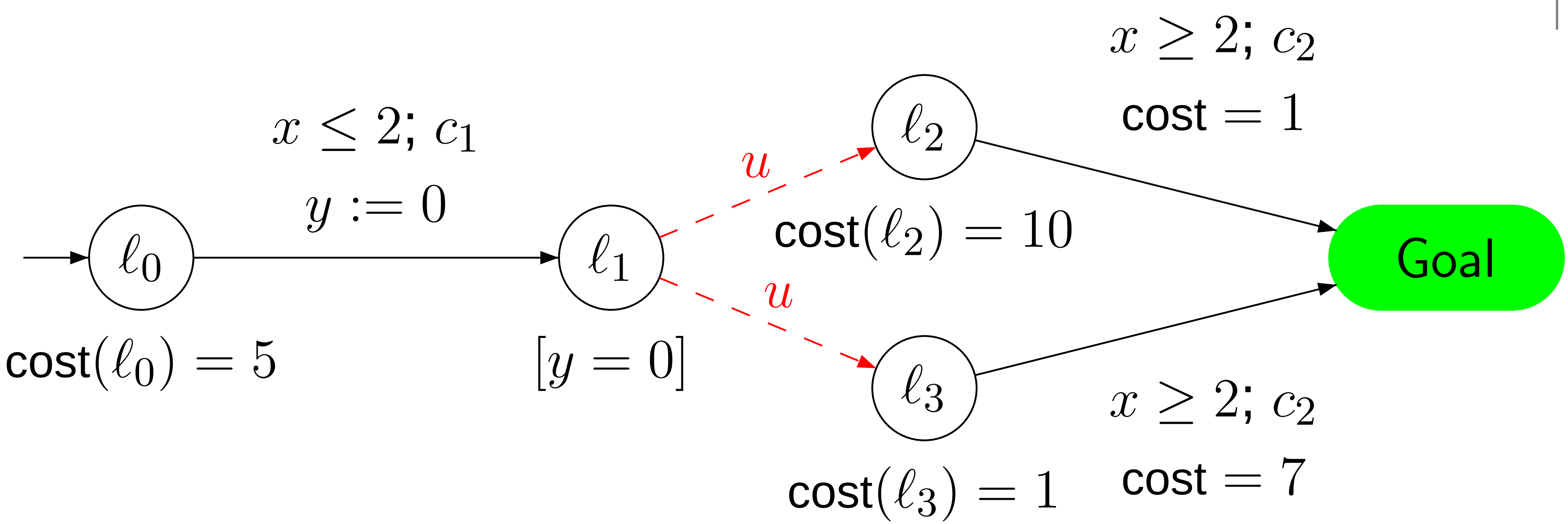
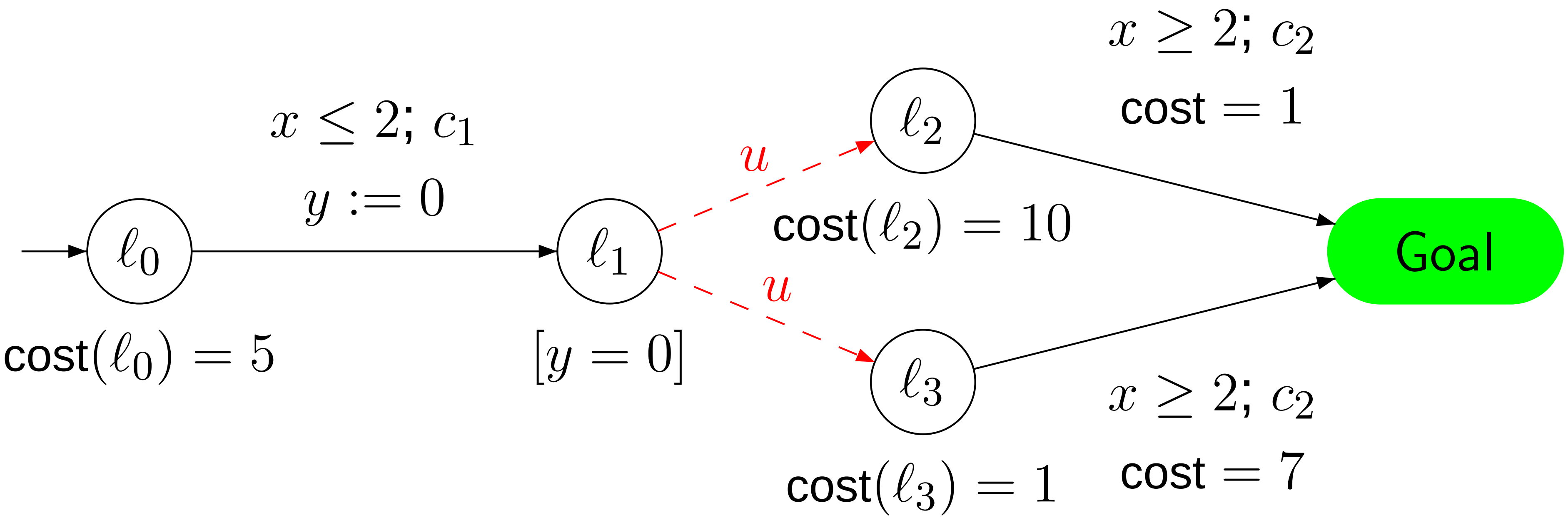
- Priced (or Weighted) Timed Automata [LBB$^+$01, ALTP01]

An Example



- Model = Game = Controller vs. Environment
- What is the best cost whatever the environment does ?

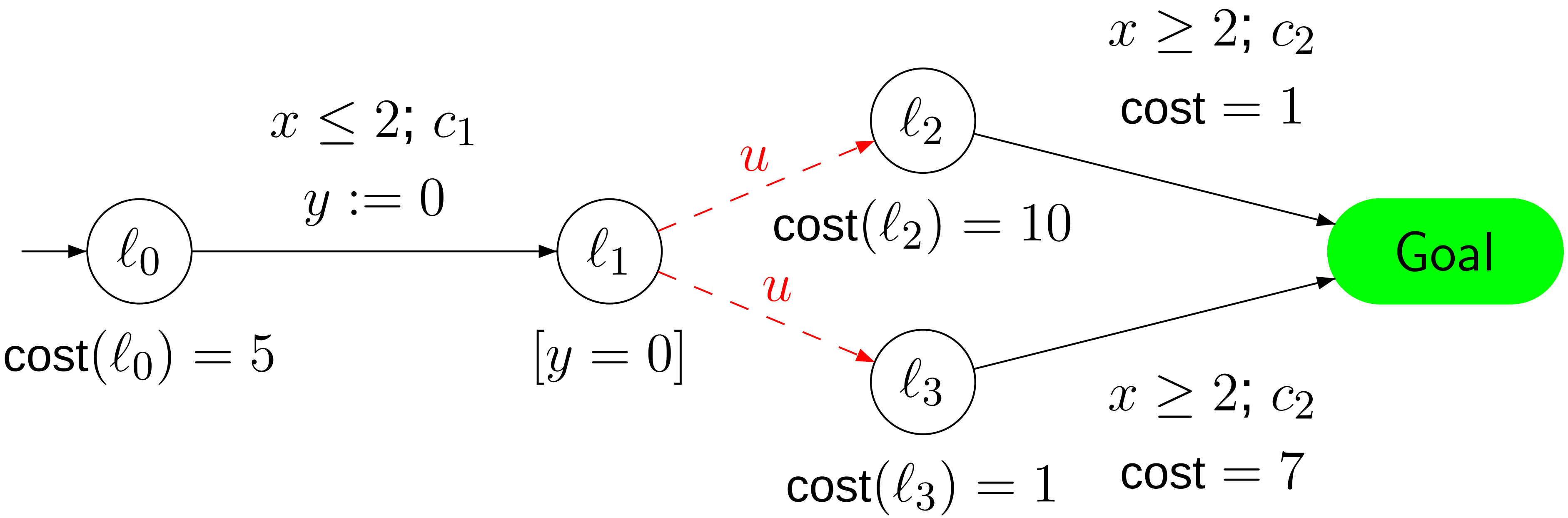
An Example



- What is the best cost whatever the environment does ?

$$\inf_{0 \leq t \leq 2} \max\{5t + 10(2-t) + 1, 5t + (2-t) + 7\}$$

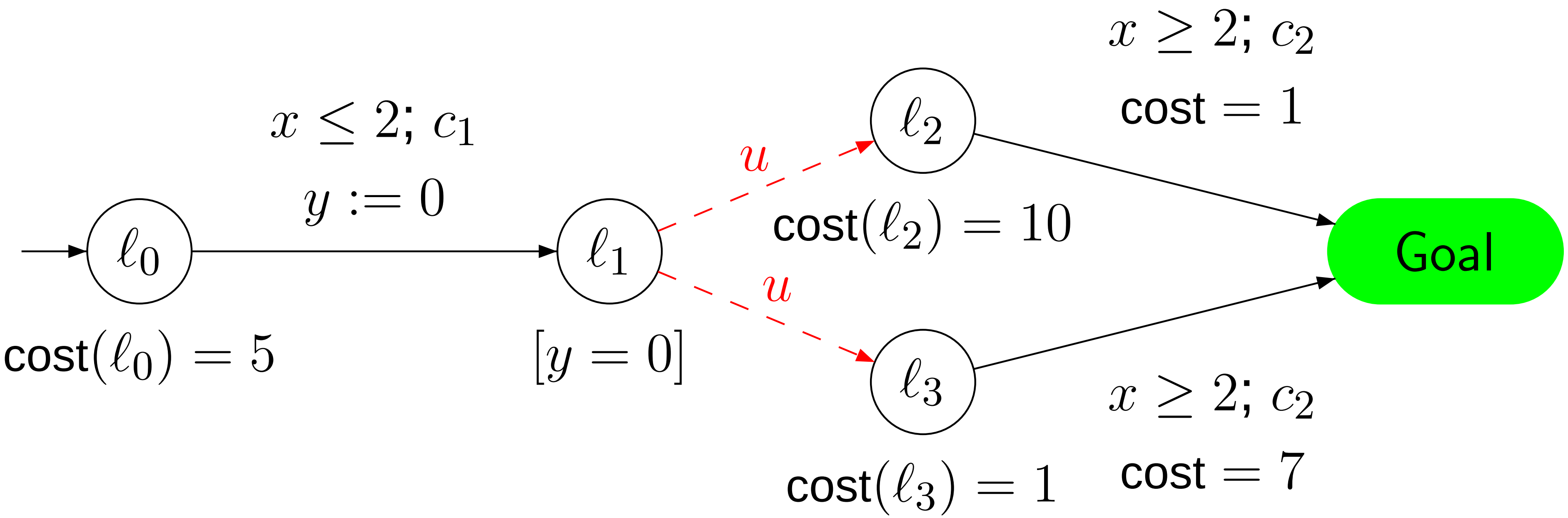
An Example



- What is the best cost whatever the environment does ?

$$\inf_{0 \leq t \leq 2} \max\{5t + 10(2-t) + 1, 5t + (2-t) + 7\} \text{ at } t = \frac{4}{3} \quad \inf = 14\frac{1}{3}$$

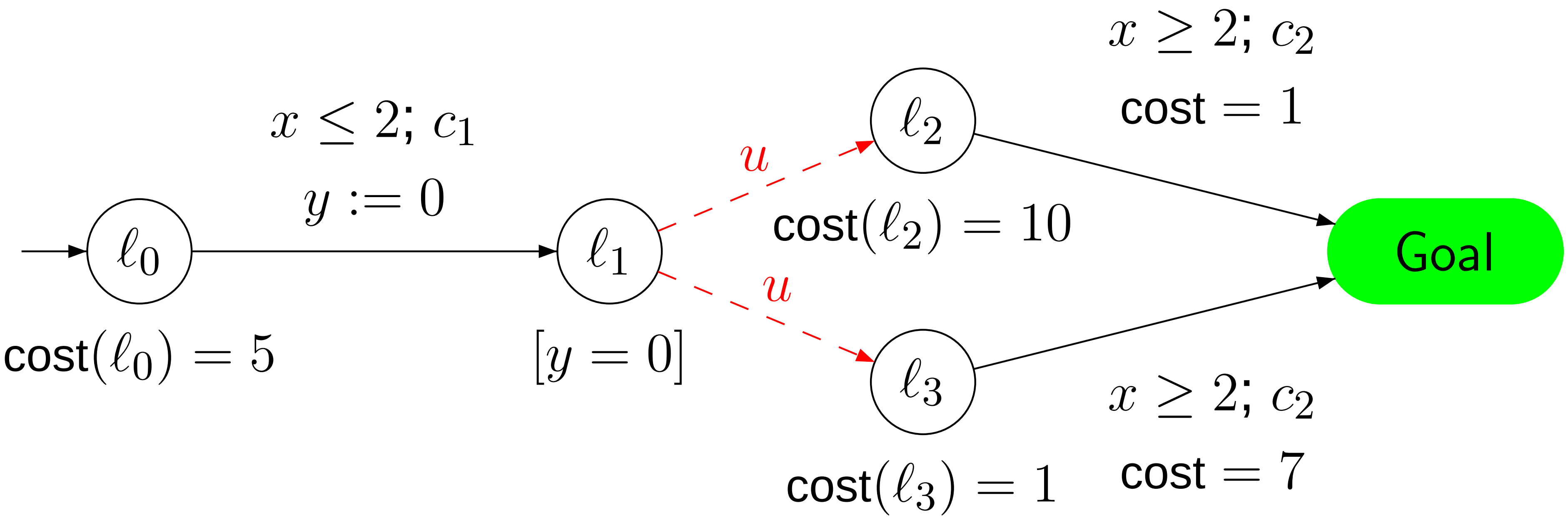
An Example



- What is the best cost whatever the environment does ?

  $\Longrightarrow 14\frac{1}{3}$

An Example



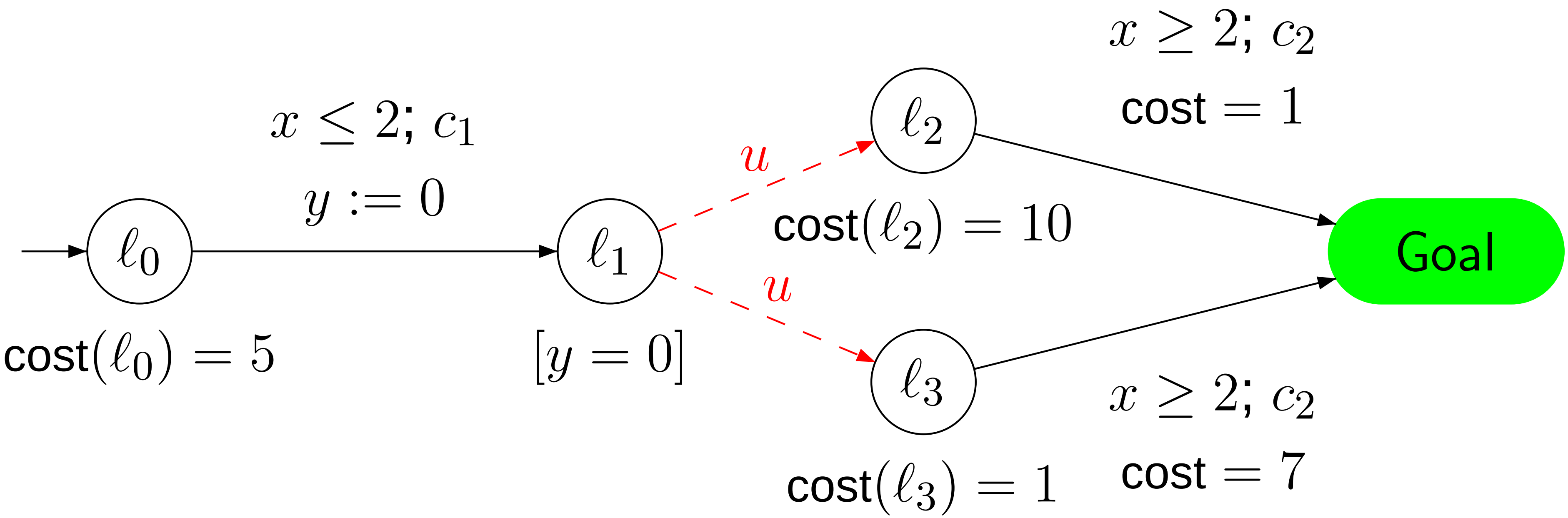
- What is the best cost whatever the environment does ?

  $\Longrightarrow 14\frac{1}{3}$

- Is there a strategy to achieve this optimal cost ?

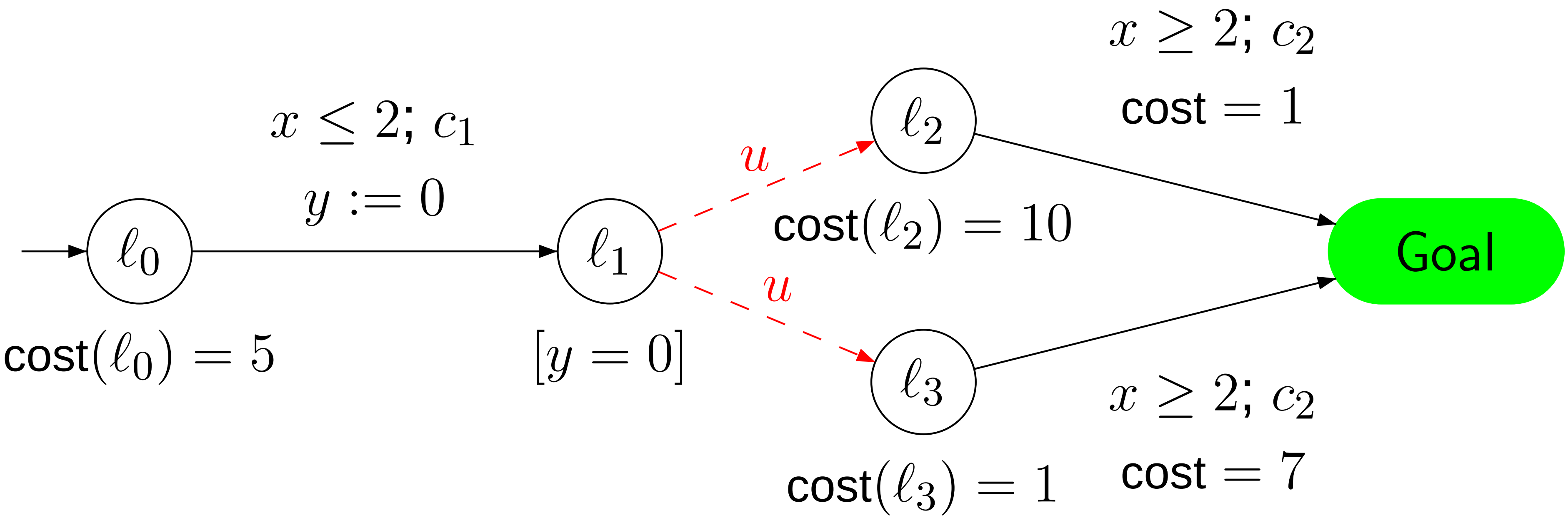
  Yes because see later

An Example



- What is the best cost whatever the environment does ?
  $\Longrightarrow 14\frac{1}{3}$
- Is there a strategy to achieve this optimal cost ?
  Yes because see later
- Can we compute such a strategy ?
  Yes: in $\ell_0, x < \frac{4}{3}$ wait then do c_1; in $\ell_{2,3}$ do c_2 when $x \geq 2$

The Problems



- Can we find an algorithm to solve these problems:
  1. What is the best cost whatever the environment does?
  2. Is there an optimal strategy?
  3. Can we compute an optimal strategy (if $\exists$)?

Related Work

- La Torre et al. [LTMM02]
  - Acyclic Priced Timed Game Automata
  - Recursive definition of optimal cost [$\Longrightarrow$ La Torre et al. Def.]
  - Computation of the infimum of the optimal cost
    $\mathrm{OptCost} = 2$ could be 2 or $2 + \varepsilon$
  - No strategy synthesis

Related Work

- La Torre et al. [LTMM02]
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- Our work:
  - Applies to Linear Hybrid Game (Automata)
  - Run-based definition of optimal cost
  - We can decide whether $\mathsf{OptCost}$ is reachable
  - We can synthetize an optimal strategy (if $\exists$)

Priced Timed Game Automata

A Timed Game Automaton (PTGA) G is a tuple $(L, \ell_0, \mathsf{Act}, X, E, \mathsf{inv}, \mathsf{cost})$ where:

- L is a finite set of locations;
- $\ell_0 \in L$ is the initial location;
- $\mathsf{Act} = \mathsf{Act}_c \cup \mathsf{Act}_u$ is the set of actions (partitioned into controllable and uncontrollable actions);
- X is a finite set of real-valued clocks;
- $E \subseteq L \times \mathcal{B}(X) \times \mathsf{Act} \times 2^X \times L$ is a finite set of transitions;
- $\mathsf{inv} : L \longrightarrow \mathcal{B}(X)$ associates to each location its invariant;

Priced Timed Game Automata

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- Priced Version: $\mathsf{cost} : L \cup E \longrightarrow \mathbb{N}$ associates to each location a cost rate and to each discrete transition a cost value.

[$\Longrightarrow$ Example]

Priced Timed Game Automata

A Priced Timed Game Automaton (PTGA) G is a tuple $(L, \ell_0, \mathsf{Act}, X, E, \mathsf{inv}, \mathsf{cost})$ where:

- L is a finite set of locations;
- $E \subseteq L \times \mathcal{B}(X) \times \mathsf{Act} \times 2^X \times L$ is a finite set of transitions;
- Priced Version: $\mathsf{cost} : L \cup E \longrightarrow \mathbb{N}$ associates to each location a cost rate and to each discrete transition a cost value. [$\Longrightarrow$ Example]
- assume that PTGA are deterministic w.r.t. controllable actions
- A reachability PTGA (RPTGA) $=$ PTGA with distinguished $\mathsf{Goal} \subseteq L$.

Configurations, Runs, Costs

- configuration: (ℓ, v) in $L \times \mathbb{R}_{\geq 0}^{X}$
- step: $t_i = (\ell_i, v_i) \xrightarrow{\alpha_i} (\ell_{i+1}, v_{i+1})$

$$\begin{cases} \alpha_i \in \mathbb{R}_{>0} \implies \ell_{i+1} = \ell_i \wedge v_{i+1} = v_i + \alpha_i \\ \alpha_i \in \mathsf{Act} \implies \exists(\ell_i, g, \alpha_i, Y, \ell_{i+1}) \in E \wedge v_i \models g \ \wedge v_{i+1} = v_i[Y] \end{cases}$$

- run $\rho = t_0 t_2 \cdots t_{n-1} \cdots$ finite of infinite sequence of t_i
- cost of a transition:

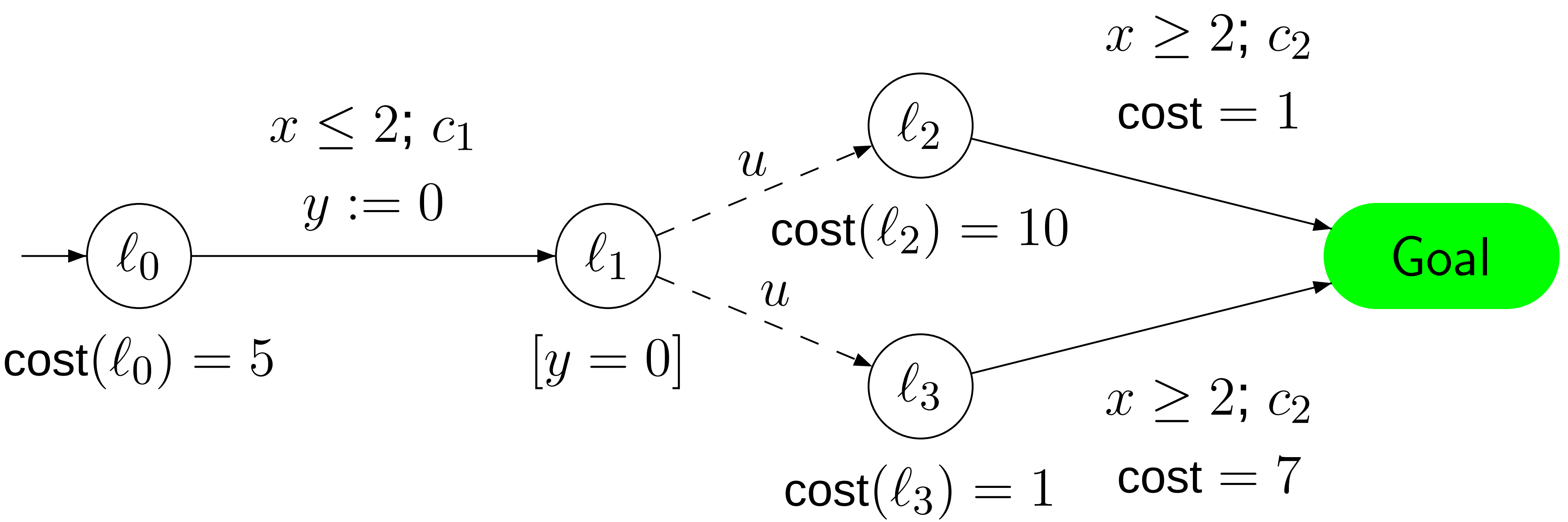
$$\begin{cases} \mathsf{Cost}(t_i) = \alpha_i.\mathsf{cost}(\ell_i) \text{ if } \alpha_i \in \mathbb{R}_{>0} \\ \mathsf{Cost}(t_i) = \mathsf{cost}((\ell_i, g, \alpha_i, Y, \ell_{i+1})) \text{ if } \alpha_i \in \mathsf{Act} \end{cases}$$

- if ρ finite $\mathsf{Cost}(\rho) = \sum_{0 \leq i \leq n-1} \mathsf{Cost}(t_i)$
- winning run if $\mathsf{States}(\rho) \cap \mathsf{Goal} \neq \emptyset$

Strategies

- strategy f over G = partial function from $\mathsf{Runs}(G)$ to $\mathsf{Act}_c \cup \{\lambda\}$.
- $\mathsf{Outcome}((\ell, v), f)$ of f from configuration (ℓ, v) in G is a subset of $\mathsf{Runs}((\ell, v), G)$ [$\Longrightarrow$ Formal Definition of Outcome]

Strategies



Example:
$$\begin{cases} f(\ell_0, x < \frac{4}{3}) = \lambda \quad f(\ell_0, \frac{4}{3} \leq x \leq 2) = c_1 \\ f(\ell_1, -) \text{ undefined} \\ f(\ell_2, x < 2) = \lambda \quad f(\ell_2, x \geq 2) = c_2 \\ f(\ell_3, x < 2) = \lambda \quad f(\ell_3, x \geq 2) = c_2 \end{cases}$$

Strategies

- strategy f over $G =$ partial function from $\mathsf{Runs}(G)$ to $\mathsf{Act}_c \cup \{\lambda\}$.
- $\mathsf{Outcome}((\ell, v), f)$ of f from configuration (ℓ, v) in G is a subset of $\mathsf{Runs}((\ell, v), G)$ [$\Longrightarrow$ Formal Definition of Outcome]
- a strategy f is winning from (ℓ, v) if

$$\mathsf{Outcome}((\ell, v), f) \subseteq \mathsf{WinRuns}((\ell, v), G)$$

- The cost of f from (ℓ, v) is

$$\mathsf{Cost}((\ell, v), f) = \sup\{\mathsf{Cost}(\rho) \mid \rho \in \mathsf{Outcome}((\ell, v), f)\}$$

Optimal Control Problems

Optimal Cost Computation Problem: compute the optimal cost one can expect from $s_0 = (\ell_0, \vec{0})$

$$\mathsf{OptCost}(s_0, G) = \inf\{\mathsf{Cost}(s_0, f) \mid f \in \mathsf{WinStrat}(s_0, G)\}$$

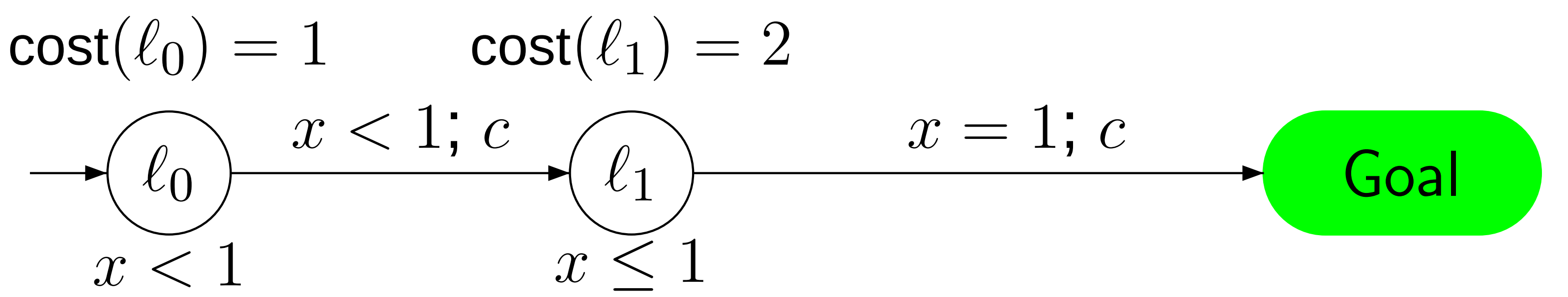
Optimal Strategy Existence Problem: determine whether the optimal cost can actually be reached

$$\exists ? f \in \mathsf{WinStrat}(s_0, G) \text{ s.t. } \mathsf{Cost}(s_0, f) = \mathsf{OptCost}(s_0, G)$$

Optimal Strategy Synthesis Problem: in case an optimal strategy exists we want to compute a witness.

Relation to La Torre et al work [LTMM02] (acyclic game):
Theorem 1: $\mathsf{OptCost}(s_0, G) = O(s_0)$ [$\Longrightarrow$ Definition of $O(q)$]

Optimal Control Problems (Cont'd-[1])

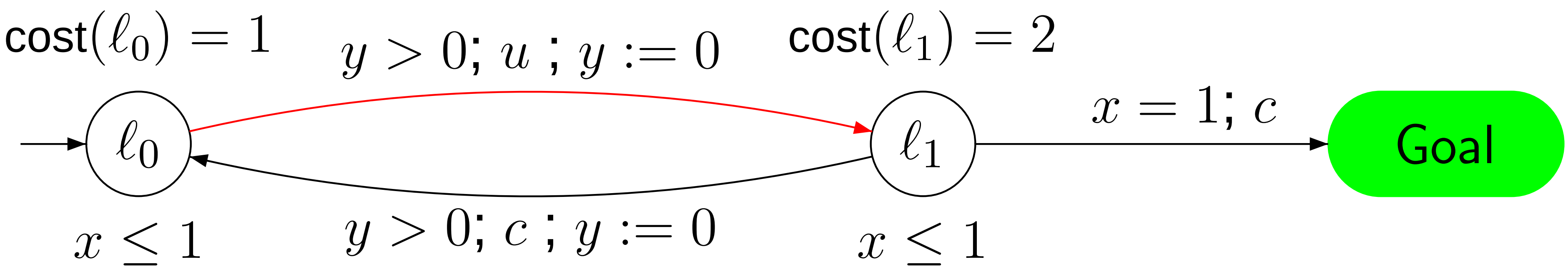


- define f_ε with $0 < \varepsilon < 1$ by:
  in ℓ_0: $f(\ell_0, x < 1 - \varepsilon) = \lambda$, $f(\ell_0, 1 - \varepsilon \leq x < 1) = c$
  in ℓ_1: $f(\ell_1, x \leq 1) = c$
  $\mathsf{Cost}(f_\varepsilon) = 1 + \varepsilon$.
- there are RPTGA for which no optimal strategy exists
- In this case there is a family of strategies f_ε such that

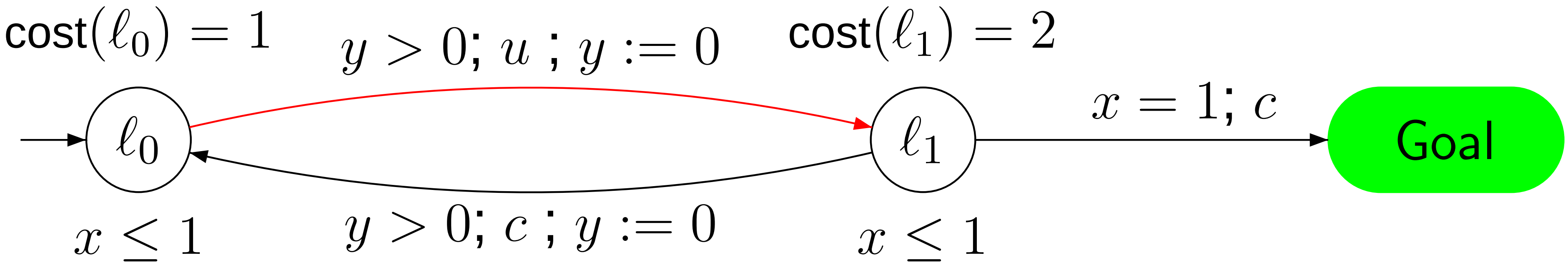
$$|\mathsf{Cost}((\ell_0, \vec{0}), f_\varepsilon) - \mathsf{OptCost}((\ell_0, \vec{0}), G)| < \varepsilon$$

- new problem: given ε, compute such an f_ε strategy.

Optimal Control Problems (Cont'd-[2])

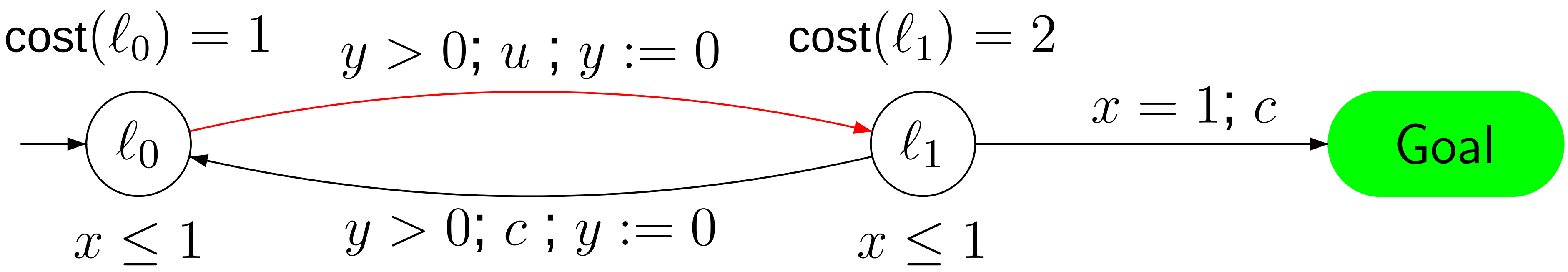


Optimal Control Problems (Cont'd-[2])



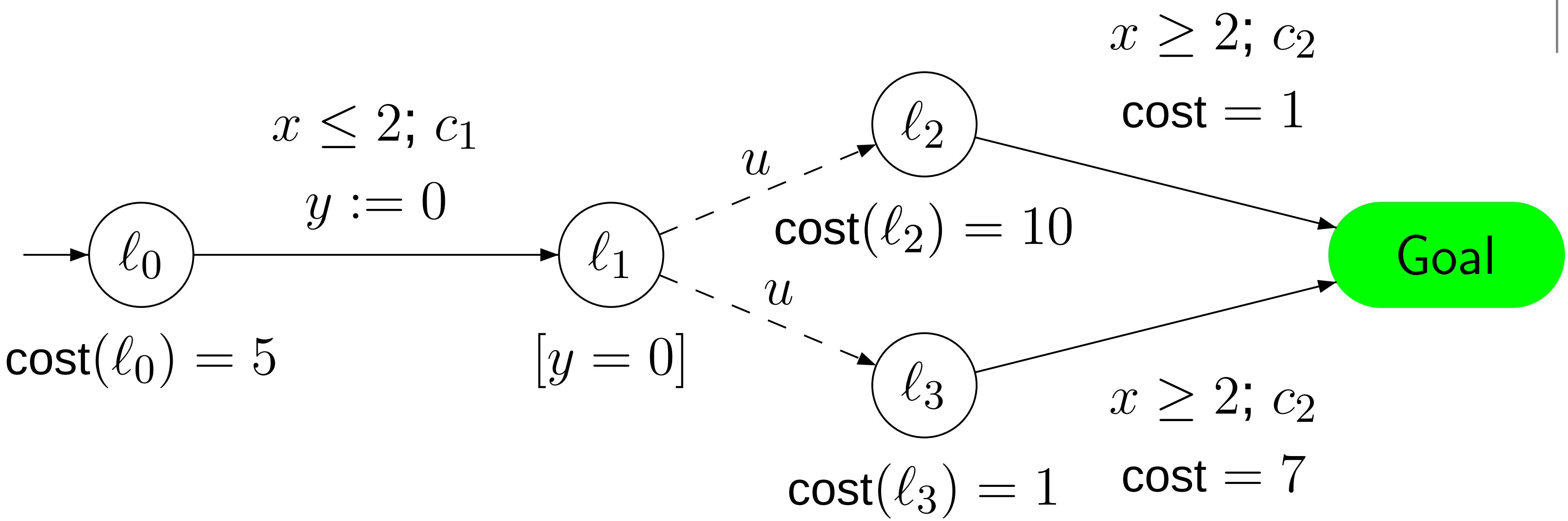
- what is the optimal cost?
- Is there an optimal strategy?

Optimal Control Problems (Cont'd-[2])



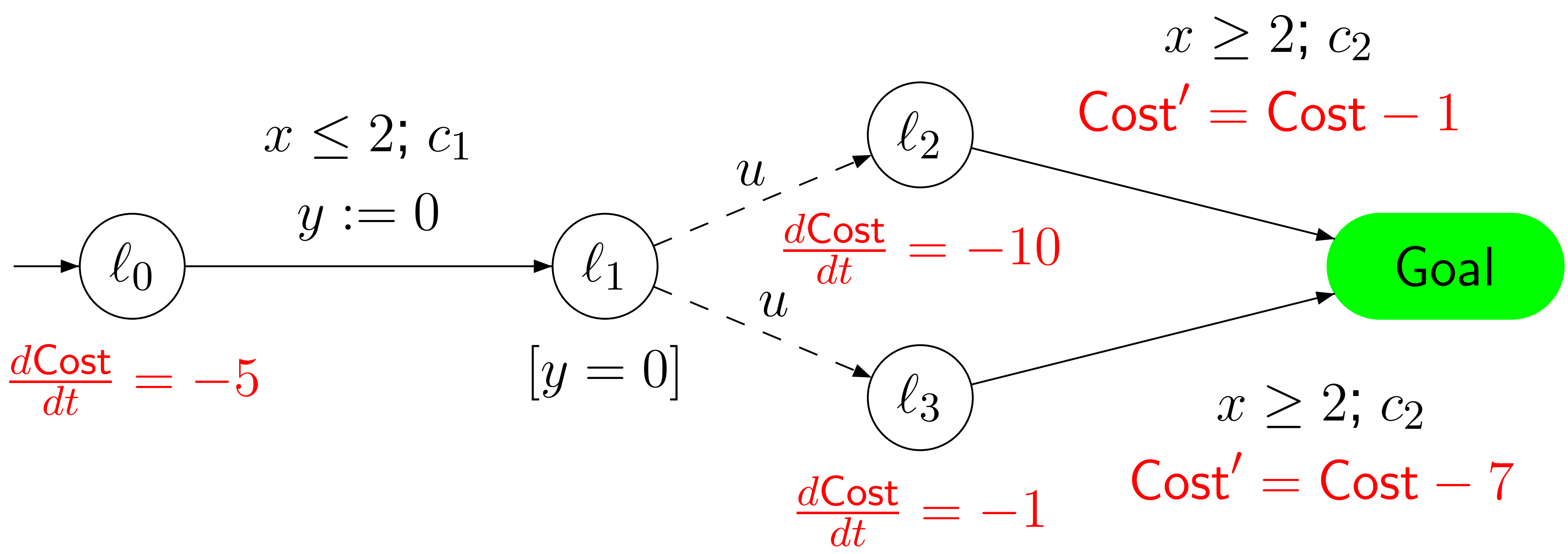
- what is the optimal cost? 2
- Is there an optimal strategy? Yes
- ... now start with 2 ... start with less than 2 ($2 - \epsilon$)

From Optimal Control to Control



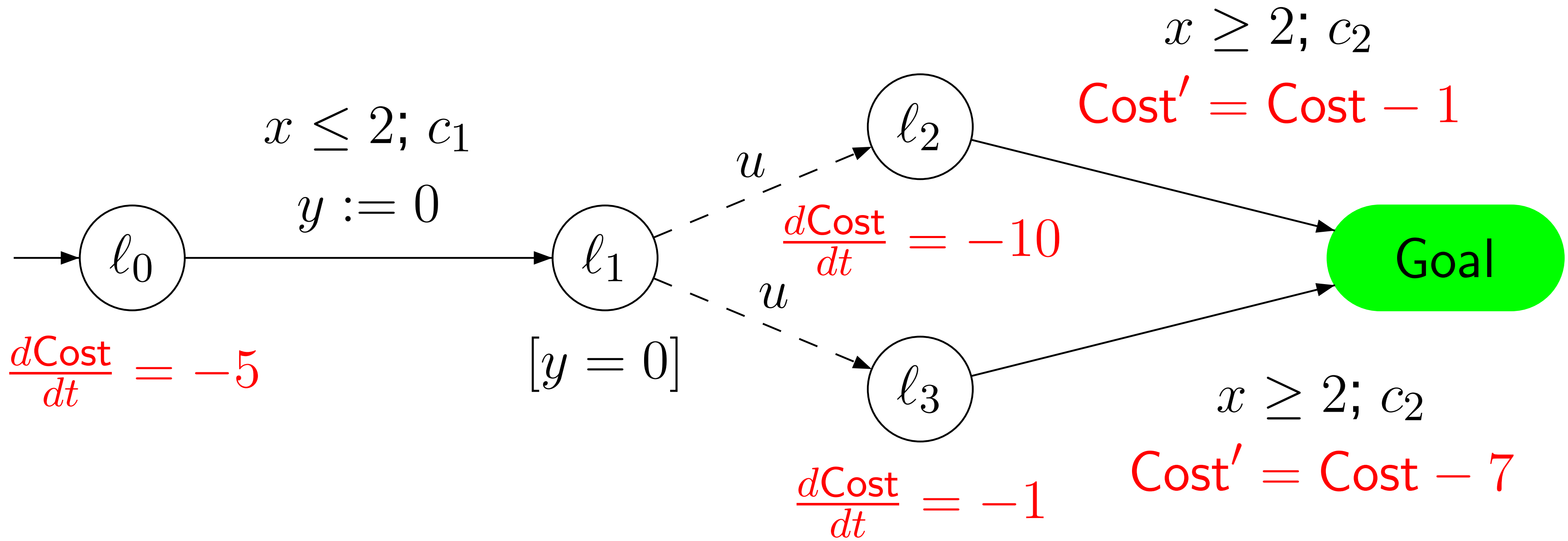
A RPTGA $\mathcal{A}$

From Optimal Control to Control



- A Linear Hybrid Game Automaton $\mathcal{H}$
- Reachability Game for $\mathcal{H}$ with goal $=$ Goal $\wedge$ Cost ≥ 0
- Optimal Cost for RPTGA $\iff$ Reachability Control on LHA

From Optimal Control to Control



Assume $\exists$ semi-algorithm CompWin s.t. $W_H = \text{CompWin}(H)$
and $W_H =$ *largest set of winning states*
Theorem 2: If CompWin terminates for H then:

- it terminates for A and $W_A \overset{\text{def}}{=} \text{CompWin}(A) = \exists cost.W_H$
- $(q, c) \in W_H \iff \exists f \in \text{WinStrat}(q, W_A)$ with $\text{Cost}(q, f) \leq c$

Results for Reachability Games

- Controllable Predecessors [MPS95, DAHM01]

$$\pi(X) = \mathsf{Pred}_t\left(X \cup \mathsf{cPred}(X), \mathsf{uPred}(\overline{X})\right)$$

[$\Longrightarrow$ Formal Def. of π]

- π preserves upwards-closed sets

$$\pi(R \wedge \mathsf{Cost} \geq k) = R' \wedge \mathsf{Cost} \geq k'$$

- W (largest) set of winning states, goal $= X_0$

$$W = \mu X.X_0 \cup \pi(X)$$

- semi-algorithm CompWin
- result of CompWin of the form $\cup_\ell(\ell, R_\ell \wedge \mathsf{Cost} \geq k)$

Computation of the Optimal Cost

Theorem 3 A a RPTGA and H its corresponding LHG.
If CompWin terminates for H

- the upward closure $\uparrow\mathsf{Cost}(\ell_0, \vec{0})$ of (the set) $\mathsf{Cost}(\ell_0, \vec{0})$ is computable
- it is either $\mathbf{cost} \geq \mathbf{k}$ (left-closed) or $\mathbf{cost} > \mathbf{k}$ (left-open) ($k \in \mathbb{Q}_{\geq 0}$)

Corollary 1 $\mathsf{OptCost}(\ell_0, \vec{0}) = \mathbf{k}$

Corollary 2
If $cost \geq k$ then $\exists$ an optimal strategy
If $cost > k$ then $\exists$ a family of strategies f_ε with $\mathsf{Cost}(f_\varepsilon) \leq k + \varepsilon$

Computing the Optimal Cost for PHGA

1. $\exists$ semi-algorithm $\mathsf{CompWin}$ for LHG
2. $W = \mathsf{CompWin}(H, \mathsf{Goal} \wedge \mathsf{Cost} \geq 0)$
3. $W_0 = W \cap (\ell_0, \vec{0})$
4. projection on Cost: $\exists(All \setminus \{\mathsf{Cost}\}).W_0$
   - if $\mathsf{Cost} \geq k$, $\mathsf{OptCost} = k$ and $\exists$ an optimal strategy
   - if $\mathsf{Cost} > k$, $\mathsf{OptCost} = k$ and $\exists$ a family of sub-optimal strategies
5. semi-algorithm for Priced Timed Hybrid Automata
6. Termination ???

Termination for RPTGA

- A a RPTGA s.t. non-zeno cost: $\exists\kappa$ s.t. every cycle in the region automaton has cost at least κ
- A is bounded *i.e.* all clocks in A are bounded

Theorem 4 CompWin terminates for H, where H is the LHG associated with A

[$\Longrightarrow$ Sketch of the Proof]

Termination for RPTGA

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Theorem 4 CompWin terminates for H, where H is the LHG associated with A [$\Longrightarrow$ Sketch of the Proof]

- Non zeno cost really needed ?
- Complexity ???

Optimal Strategy Synthesis

- $\mathcal{S}$ algorithm for synthetizing strategies for reachability timed games ? see [BCFG04] ...
- use $\mathcal{S}$ on the LHG H: strategies are cost-dependent

Theorem 5 If $\mathcal{S}$ terminates and $\exists$ an optimal strategy we can compute a witness (cost-dependent)

Optimal Strategy Synthesis

- $\mathcal{S}$ algorithm for synthetizing strategies for reachability timed games ? see [BCFG04] ...
- use $\mathcal{S}$ on the LHG H: strategies are cost-dependent

Theorem 5 If $\mathcal{S}$ terminates and $\exists$ an optimal strategy we can compute a witness (cost-dependent)

- assume a RPTGA A is bounded, non zeno cost
- W is the set of winning states in the LHG H
- $W = \cup_{\ell \in L}((\ell, R_\ell \wedge \mathsf{Cost} \geq k_\ell))$

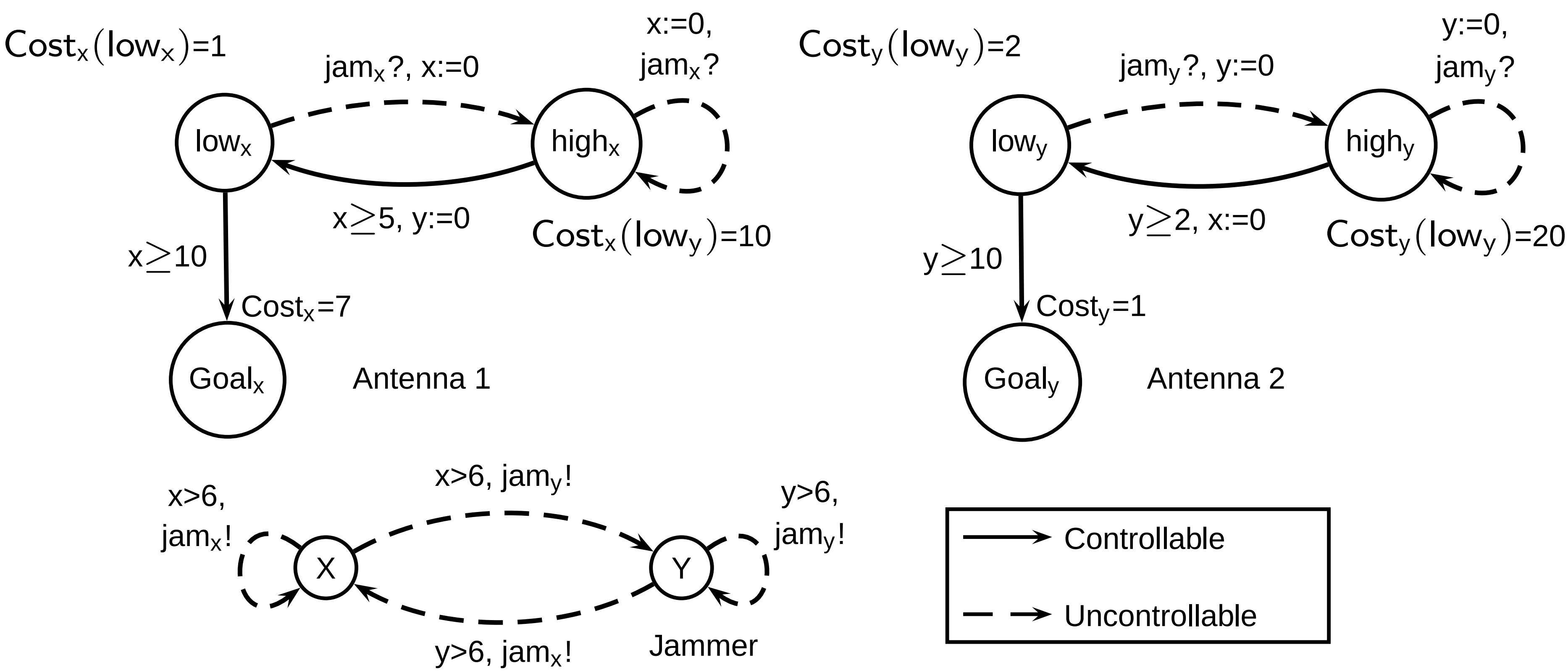
Theorem 6 [State-based Strategies] $W_A = \mathsf{CompWin}(A)$. Then:

$$\exists f \in \mathsf{WinStrat}(A) \text{ s.t. } \mathsf{Cost}((\ell, v), f) = \mathsf{OptCost}(\ell, v)\, \forall (\ell, v) \in W_A$$

Implementation

- computation of optimal cost and optimal strategies (if ∃) implemented in HyTech (Demo ?)
- an cyclic example:

[⟹ See the strategy]



Conclusion & Future Work

Current State of Our Work

- Semi-algorithm for computing the optimal cost for LHG
- in case it terminates:
  - decide if ∃ optimal strategy
  - compute an optimal (cost-independent) strategy
- Implementation in HyTech

Open Problems

- Time Optimal Control – Decidability issues
- maximal class for which CompWin terminates

Future Work

- compute f_ε strategies
- safety games . . .

References

[AD94] R. Alur and D. Dill. A theory of timed automata. *Theoretical Computer Science (TCS)*, 126(2):183–235, 1994. 3

[ALTP01] R. Alur, S. La Torre, and G. J. Pappas. Optimal paths in weighted timed automata. In *Proc. 4th Int. Work. Hybrid Systems: Computation and Control (HSCC'01)*, volume 2034 of *LNCS*, pages 49–62. Springer, 2001. 3

[AM99] E. Asarin and O. Maler. As soon as possible: Time optimal control for timed automata. In *Proc. 2nd Int. Work. Hybrid Systems: Computation and Control (HSCC'99)*, volume 1569 of *LNCS*, pages 19–30. Springer, 1999. 3

[AMPS98] E. Asarin, O. Maler, A. Pnueli, and J. Sifakis. Controller synthesis for timed automata. In *Proc. IFAC Symposium on System Structure and Control*, pages 469–474. Elsevier Science, 1998. 3

[BCFG04] P. Bouyer, F. Cassez, E. Fleury, and Larsen K. G. Optimal strategies in priced timed game automata. BRICS Report Series, Basic Research In Computer Science, Denmark, February 2004. 15

References

[DAHM01] L. De Alfaro, T. A. Henzinger, and R. Majumdar. Symbolic algorithms for infinite-state games. In *Proc. 12th International Conference on Concurrency Theory (CONCUR'01)*, volume 2154 of *LNCS*, pages 536–550. Springer, 2001. 11

[LBB+01] K. G. Larsen, G. Behrmann, E. Brinksma, A. Fehnker, T. Hune, P. Pettersson, and J. Romijn. As cheap as possible: Efficient cost-optimal reachability for priced timed automata. In *Proc. 13th International Conference on Computer Aided Verification (CAV'01)*, volume 2102 of *LNCS*, pages 493–505. Springer, 2001. 3

[LTMM02] S. La Torre, S. Mukhopadhyay, and A. Murano. Optimal-reachability and control for acyclic weighted timed automata. In *Proc. 2nd IFIP International Conference on Theoretical Computer Science (TCS 2002)*, volume 223 of *IFIP Conference Proceedings*, pages 485–497. Kluwer, 2002. 5, 9

[MPS95] O. Maler, A. Pnueli, and J. Sifakis. On the synthesis of discrete controllers for timed systems. In *Proc. 12th Annual Symposium on Theoretical Aspects of Computer Science (STACS'95)*, volume 900, pages 229–242. Springer, 1995. 3, 11

Recursive Definition of Optimal Cost

Let G be a RPTG. Let O be the function from Q to $\mathbb{R}_{\geq 0} \cup \{+\infty\}$ that is the least fixed point of the following functional:

$$O(q)? \quad q \xrightarrow{t,p} q' \quad \max \left\{ \right.$$

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$$O(q)? \quad q \xrightarrow{t,p} q' \quad \max \left\{ \begin{array}{l} \min\left(\left(\min_{\substack{q' \xrightarrow{c,p'} q'' \\ c \in \mathsf{Act}_c}} p + p' + O(q'') \right), p + O(q') \right) \\ \\ \\ \end{array} \right.$$

- Controllable actions in q'

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- Controllable actions in q'
- Uncontrollable actions before q'

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$$O(q) = \inf_{\substack{q \xrightarrow{t,p} q' \\ t \in \mathbb{R}_{\geq 0}}} \max \begin{cases} \min\left(\left(\min_{\substack{q' \xrightarrow{c,p'} q'' \\ c \in \mathsf{Act}_c}} p + p' + O(q'')\right), p + O(q')\right) \\ \sup_{\substack{q \xrightarrow{t',p'} q'' \\ t' \leq t}} \max_{\substack{q'' \xrightarrow{u,p''} q''' \\ u \in \mathsf{Act}_u}} p' + p'' + O(q''') \end{cases}$$

- Controllable actions in q'
- Uncontrollable actions before q'
- Minimize over t

Outcome

Let $G = (L, \ell_0, \mathsf{Act}, X, E, \mathsf{inv}, \mathsf{cost})$ be a (R)PTGA and f a strategy over G. The outcome $\mathsf{Outcome}((\ell, v), f)$ of f from configuration (ℓ, v) in G is the subset of $\mathsf{Runs}((\ell, v), G)$ defined inductively by:

- $(\ell, v) \in \mathsf{Outcome}((\ell, v), f)$,
- if $\rho \in \mathsf{Outcome}((\ell, v), f)$ then $\rho' = \rho \xrightarrow{e} (\ell', v') \in \mathsf{Outcome}((\ell, v), f)$ if $\rho' \in \mathsf{Runs}((\ell, v), G)$ and one of the following three conditions hold:
  1. $e \in \mathsf{Act}_u$,
  2. $e \in \mathsf{Act}_c$ and $e = f(\rho)$,
  3. $e \in \mathbb{R}_{\geq 0}$ and $\forall 0 \leq e' < e, \exists (\ell'', v'') \in (L \times \mathbb{R}^X_{\geq 0})$ s.t. $last(\rho) \xrightarrow{e'} (\ell'', v'') \wedge f(\rho \xrightarrow{e'} (\ell'', v'')) = \lambda$.
- an infinite run ρ is in $\in \mathsf{Outcome}((\ell, v), f)$ if all the finite prefixes of ρ are in $\mathsf{Outcome}((\ell, v), f)$.

[$\Longrightarrow$ Back to Strategies]

π Operator

- (Un)Controllable Predecessors

$$\mathsf{Pred}^a(X) = \{q \in Q \mid q \xrightarrow{a} q', q' \in X\}$$

$$\mathsf{cPred}(X) = \bigcup_{c \in \mathsf{Act}_c} \mathsf{Pred}^c(X) \quad \mathsf{uPred}(X) = \bigcup_{u \in \mathsf{Act}_u} \mathsf{Pred}^u(X)$$

- Safe Time Predecessors $\mathsf{Pred}_t(X, Y)$

$$= \{q \in Q \mid \exists \delta \in \mathbb{R}_{\geq 0} \mid q \xrightarrow{\delta} q',\ q' \in X \wedge \mathsf{Post}_{[0,\delta]}(q) \subseteq \overline{Y}\}$$

$$\mathsf{Post}_{[0,\delta]}(q) = \{q' \in Q \mid \exists t \in [0, \delta] \mid q \xrightarrow{t} q'\}$$

- π Operator (uncontrollable actions “cannot win”):

$$\pi(X) = \mathsf{Pred}_t\left(X \cup \mathsf{cPred}(X), \mathsf{uPred}(\overline{X})\right)$$

π Operator

- (Un)Controllable Predecessors

$$\mathsf{Pred}^a(X) = \{q \in Q \mid q \xrightarrow{a} q', q' \in X\}$$

$$\mathsf{cPred}(X) = \bigcup_{c \in \mathsf{Act}_c} \mathsf{Pred}^c(X) \quad \mathsf{uPred}(X) = \bigcup_{u \in \mathsf{Act}_u} \mathsf{Pred}^u(X)$$

- Safe Time Predecessors $\mathsf{Pred}_t(X, Y)$

$$= \{q \in Q \mid \exists \delta \in \mathbb{R}_{\geq 0} \mid q \xrightarrow{\delta} q', \; q' \in X \wedge \mathsf{Post}_{[0,\delta]}(q) \subseteq \overline{Y}\}$$

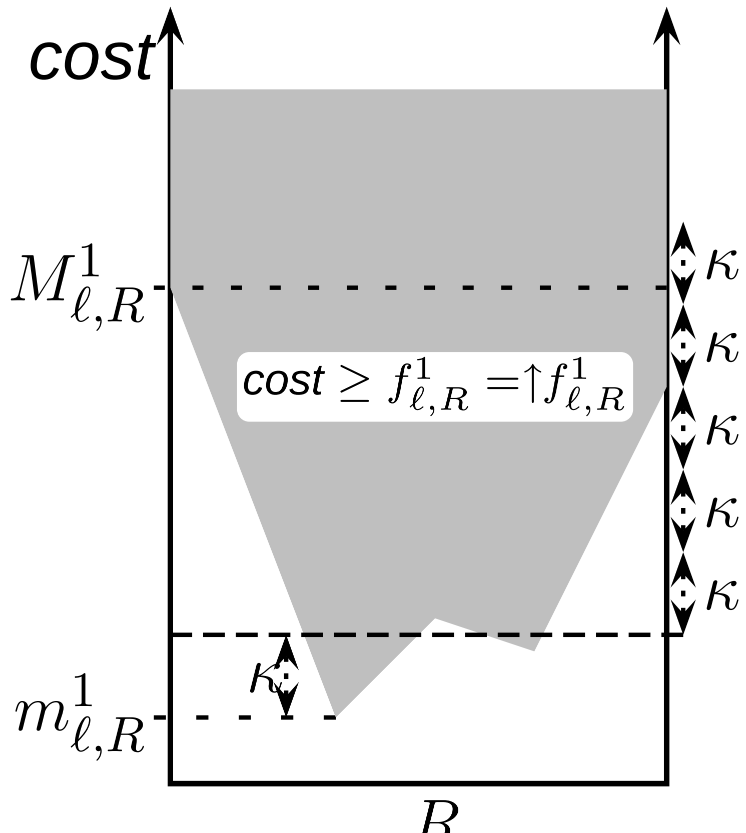
$$\mathsf{Post}_{[0,\delta]}(q) = \{q' \in Q \mid \exists t \in [0, \delta] \mid q \xrightarrow{t} q'\}$$

- π': uncontrollable actions sometimes can win:

$$\pi'(X) = \mathsf{Pred}_t\left(X \cup \mathsf{cPred}(X) \cup (\mathsf{uPred}(X) \cap STOP), \mathsf{uPred}(\overline{X})\right)$$

Termination Criterion for RPTGA

- R is a (bounded) region of the region automaton (RA)
- every cycle in the RA costs at least κ



[$\Longrightarrow$ Back to Termination]

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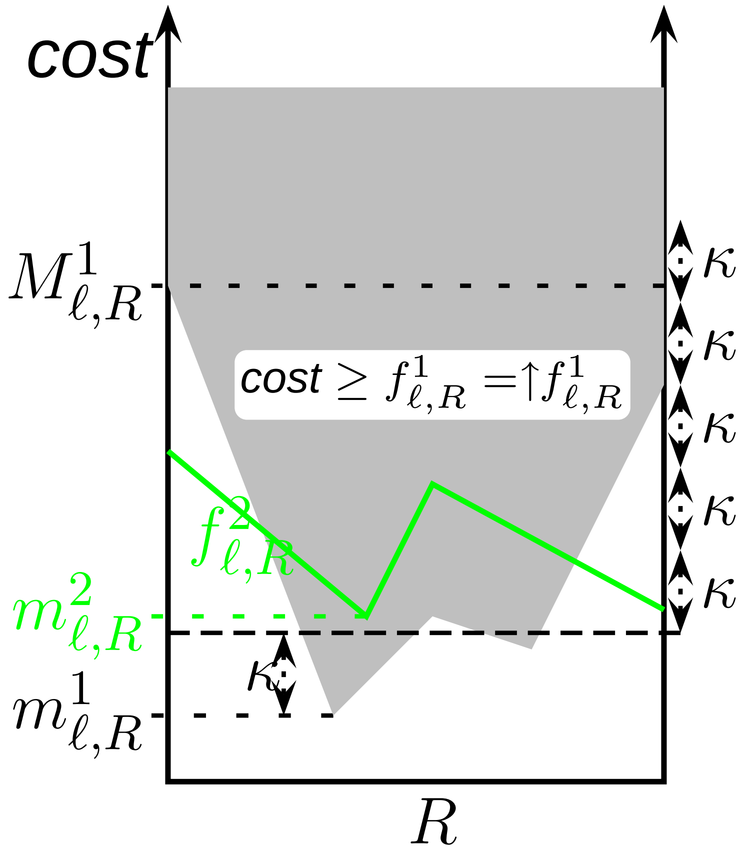

[$\Longrightarrow$ Back to Termination]

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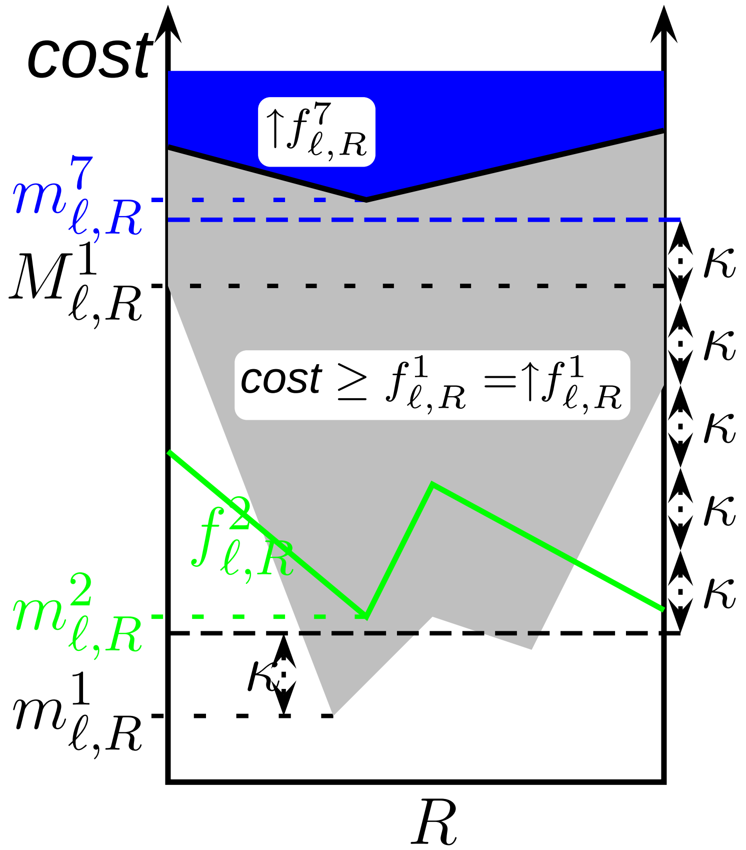

[$\Longrightarrow$ Back to Termination]

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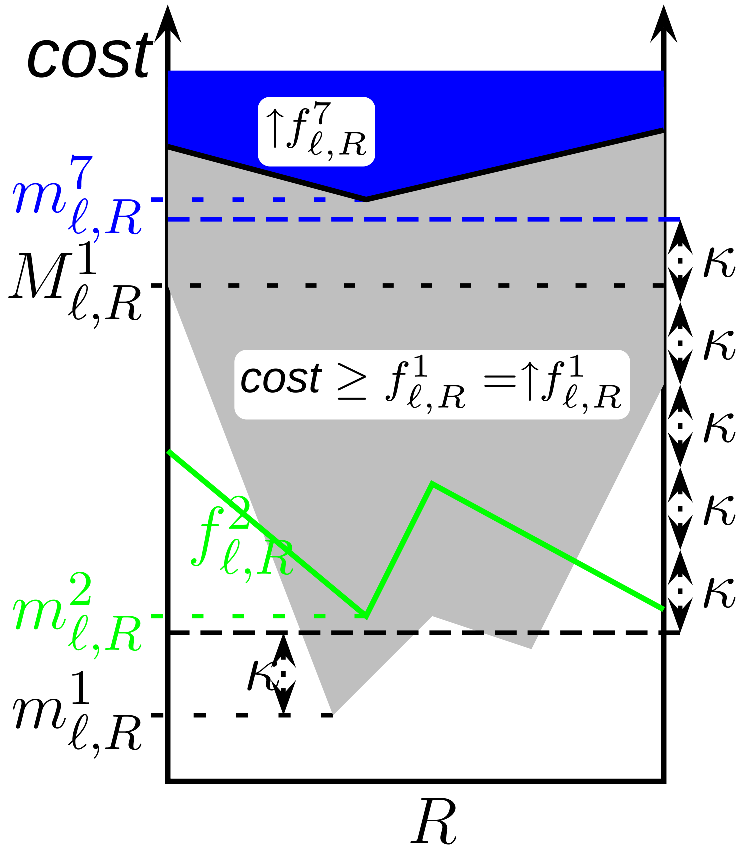

[$\Longrightarrow$ Back to Termination]

Optimal Strategy for the Mobile Phone

Optimal cost is 109