Inside Keccak

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Keccak & SHA-3 Day
Université Libre de Bruxelles
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Outline

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1. Defining KECCAK
2. Differential and linear trail propagation
3. Alignment
4. Bounding differential and linear trail weights
5. The kernel

The beginning

- SUBTERRANEAN: Daemen (1991)
  - variable-length input and output
  - hashing and stream cipher
  - round function interleaved with input/output
- STEPRIGHTUP: Daemen (1994)
- PANAMA: Daemen and Clapp (1998)
- RADIOGATÚN: Bertoni, Daemen, Peeters and VA (2006)
  - experiments did not inspire confidence in RADIOGATÚN
  - neither did third-party cryptanalysis
    [Bouillaguet, Fouque, SAC 2008] [Fuhr, Peyrin, FSE 2009]
  - NIST SHA-3 deadline approaching ...
  - U-turn: design a sponge with strong permutation f
- KECCAK (2008)

Designing the permutation KECCAK-f

Our mission

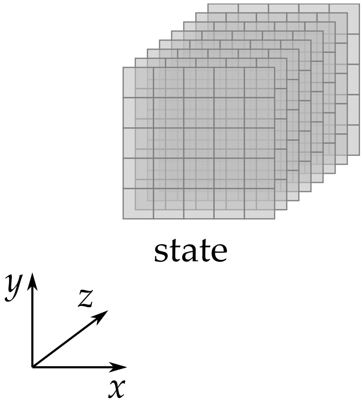
To design a permutation called KECCAK-f that cannot be distinguished from a random permutation.

- Like a block cipher
  - sequence of identical rounds
  - round function that is nonlinear and has good diffusion
- ...but not quite
  - no need for key schedule
  - round constants instead of round keys
  - inverse permutation need not be efficient

KECCAK

- Instantiation of a *sponge function*
- the permutation KECCAK-f
  - 7 permutations: $b \in \{25, 50, 100, 200, 400, 800, 1600\}$
- Security-speed trade-offs using the same permutation, e.g.,
  - SHA-3 instance: $r = 1088$ and $c = 512$
    - permutation width: 1600
    - security strength 256: post-quantum sufficient
  - Lightweight instance: $r = 40$ and $c = 160$
    - permutation width: 200
    - security strength 80: same as SHA-1

The state: an array of $5 \times 5 \times 2^\ell$ bits



- 5×5 lanes, each containing 2^ℓ bits (1, 2, 4, 8, 16, 32 or 64)
- (5×5)-bit slices, 2^ℓ of them

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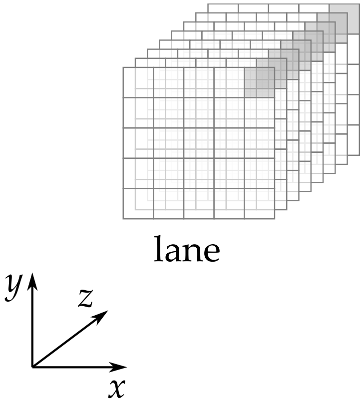

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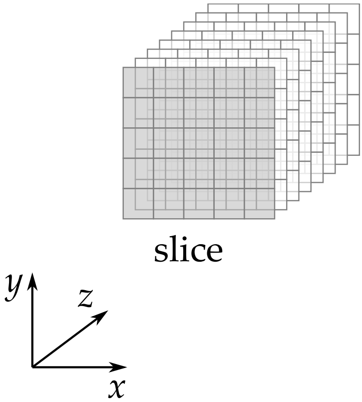

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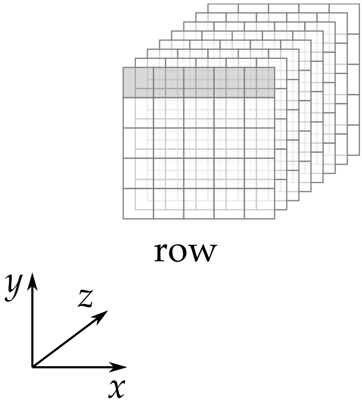

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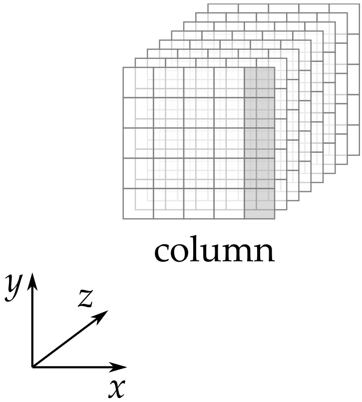

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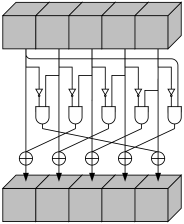
χ, the nonlinear mapping in KECCAK-f

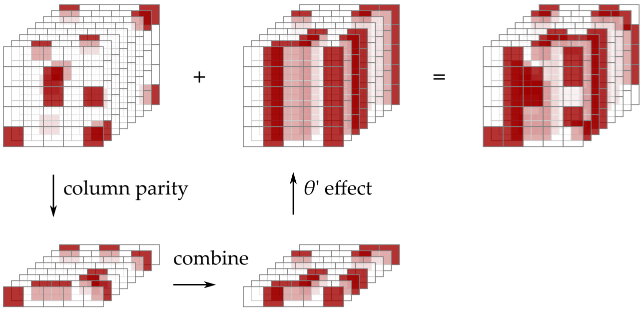


- "Flip bit if neighbors exhibit 01 pattern"
- Operates independently and in parallel on 5-bit rows
- Algebraic degree 2, inverse has degree 3
- LC/DC propagation properties easy to describe and analyze

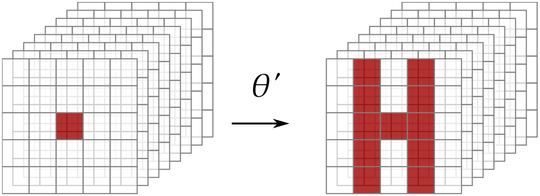
θ', a first attempt at mixing bits

- Compute parity $c_{x,z}$ of each column
- Add to each cell parity of neighboring columns:

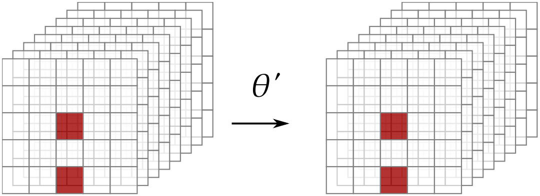
$$b_{x,y,z} = a_{x,y,z} \oplus c_{x-1,z} \oplus c_{x+1,z}$$



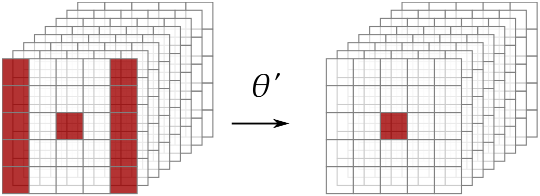
Diffusion of θ'



Diffusion of θ' (kernel)



Diffusion of the inverse of θ'

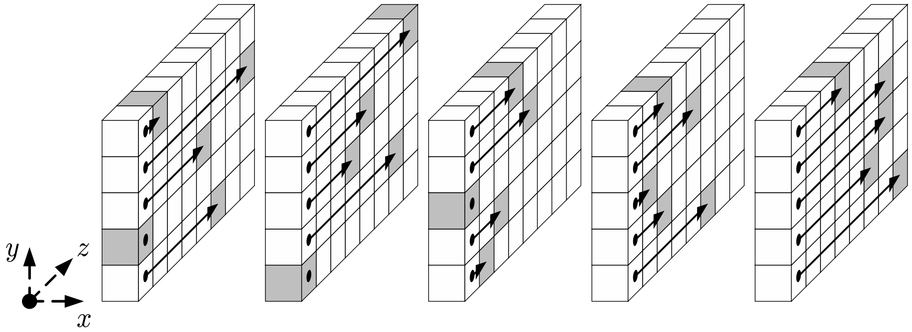


ρ for inter-slice dispersion

- We need diffusion between the slices …
- ρ: cyclic shifts of lanes with offsets

$$i(i+1)/2 \bmod 2^{\ell}$$

- Offsets cycle through all values below 2^{ℓ}

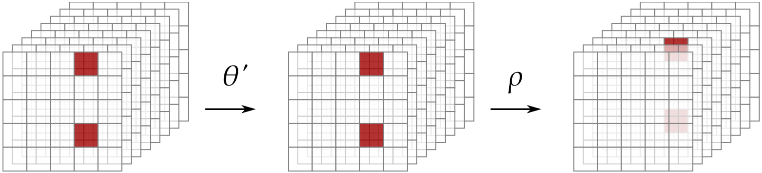


ι to break symmetry

- XOR of round-dependent constant to lane in origin
- Without ι, the round mapping would be symmetric
  - invariant to translation in the z-direction
- Without ι, all rounds would be the same
  - susceptibility to *slide* attacks
  - defective cycle structure
- Without ι, we get simple fixed points (000 and 111)

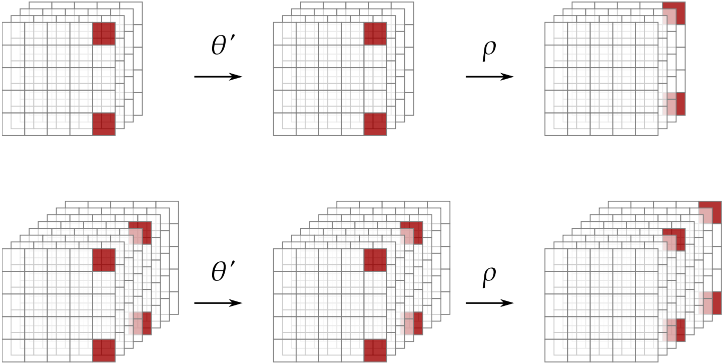
A first attempt at KECCAK-f

- Round function: $\mathrm{R} = \iota \circ \rho \circ \theta' \circ \chi$
- Problem: low-weight periodic trails by chaining:



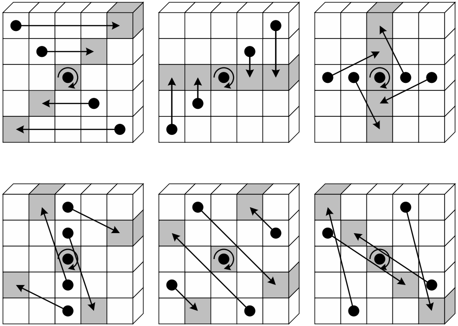
  - χ: may propagate unchanged
  - θ': propagates unchanged, because all column parities are 0
  - ρ: in general moves active bits to different slices ...
  - ...but not always

The Matryoshka property



- Patterns in Q' are z-periodic versions of patterns in Q

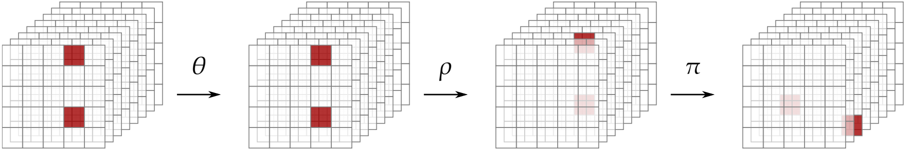
π for disturbing horizontal/vertical alignment



$$a_{x,y} \leftarrow a_{x',y'} \text{ with } \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

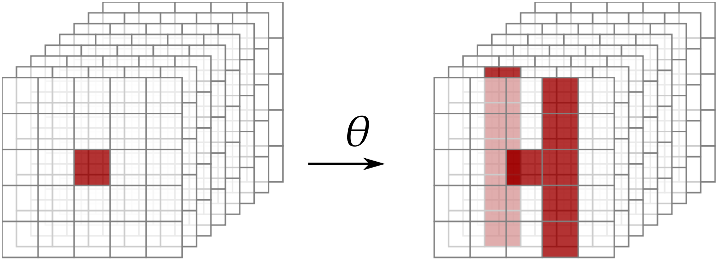
A second attempt at Keccak-f

- Round function: $\mathrm{R} = \iota \circ \pi \circ \rho \circ \theta' \circ \chi$
- Solves problem encountered before:



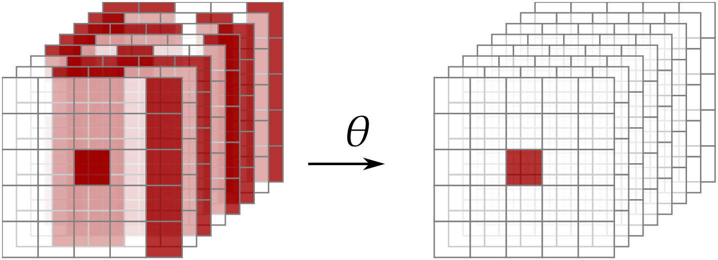
- π moves bits in same column to different columns!

Tweaking θ' to θ



$$b_{x,y,z} = a_{x,y,z} \oplus c_{x-1,z} \oplus c_{x+1,z-1}$$

Inverse of θ



- Diffusion from single-bit output to input very high
- Increases resistance against LC/DC and algebraic attacks

KECCAK-f summary

Round function

$$\text{round} = \iota \circ \chi \circ \pi \circ \rho \circ \theta$$

- Number of rounds: $12 + 2\ell$
  - KECCAK-$f[25]$ has 12 rounds
  - KECCAK-$f[1600]$ has 24 rounds

Design decisions behind KECCAK-f

- Ability to control propagation of differences or linear masks
  - Differential/linear trail analysis
  - Lower bounds for trail weights
  - Alignment and trail clustering
  - $\Rightarrow$ This shaped θ, π and ρ
- Algebraic properties
  - Distribution of $\#$ terms of certain degrees
  - Ability of solving certain problems (CICO) algebraically
  - Zero-sum distinguishers (third party)
  - $\Rightarrow$ This determined the number of rounds
- Analysis of symmetry properties
  $\Rightarrow$ This shaped ι

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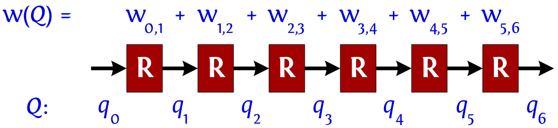
Differential and linear trails in iterated mappings



- Differential trail: sequence of differences

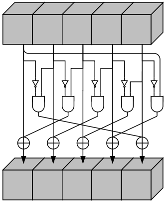
$$\text{weight} = -\log_2(\text{fraction of pairs})$$

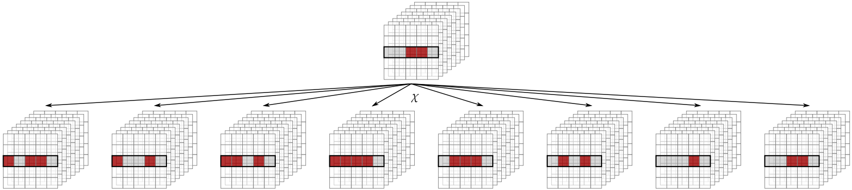
- Linear trail: sequence of linear masks

$$\text{weight} = -2\log_2(\text{correlation contribution})$$

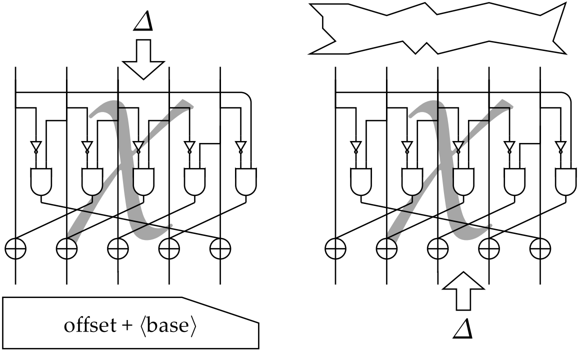
Non-linear mapping χ

- Transforms each **row** independently
- E.g., a difference going through χ
  - Output: affine space



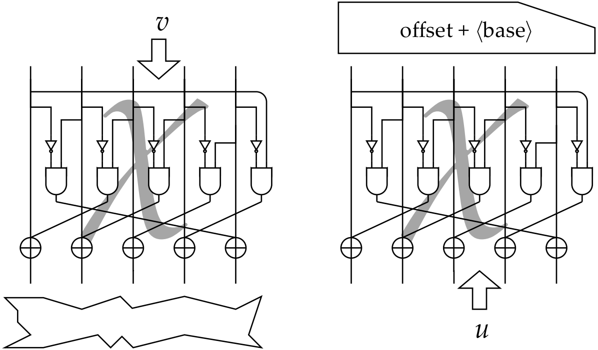


Propagating differences through χ



- The propagation weight...
  - ... is determined by input difference only;
  - ... is the size of the affine base;
  - ... is the number of affine conditions.

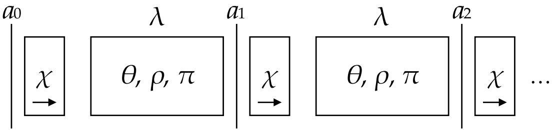
Propagating linear masks through χ



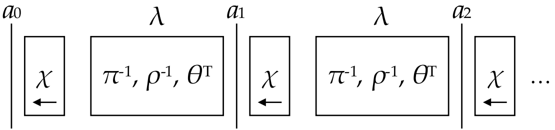
- The propagation weight...
  - ... is determined by output mask only;
  - ... is the size of the affine base.

Differential and linear trails in KECCAKTOOLS

- KECCAKTOOLS
  - *A set of documented C++ classes to help analyze* KECCAK
    Freely available on http://keccak.noekeon.org
  - Implements differential and linear trail propagation
- KeccakFPropagation works in "affine" direction:
  - Differential trails



  - Linear trails: forward propagation means backwards in time



Outline

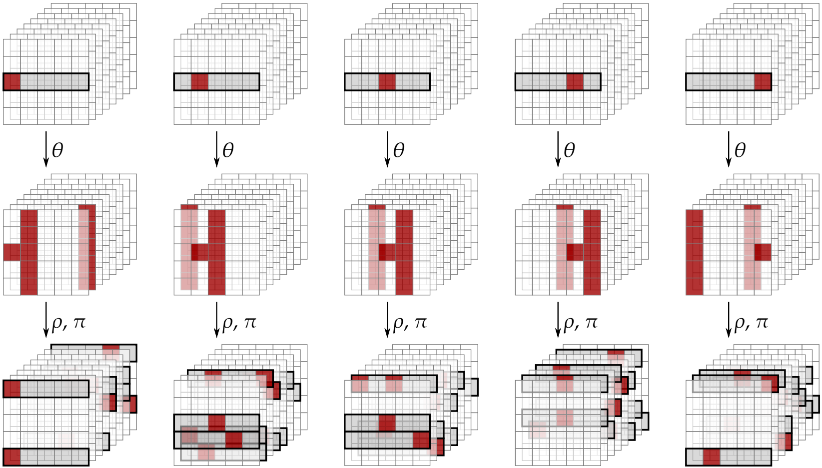
Difference propagation in RIJNDAEL

- Differential trail (fully specified)
  - Deterministic propagation through MixColumns, ShiftRows and AddRoundKey
  - Branching through SubBytes
- Truncated diff. trail specifying active/passive s-boxes
  - Deterministic propagation through SubBytes, ShiftRows and AddRoundKey
  - Branching through MixColumns
    - Sometimes deterministic: 1 byte $\rightarrow$ 4 bytes

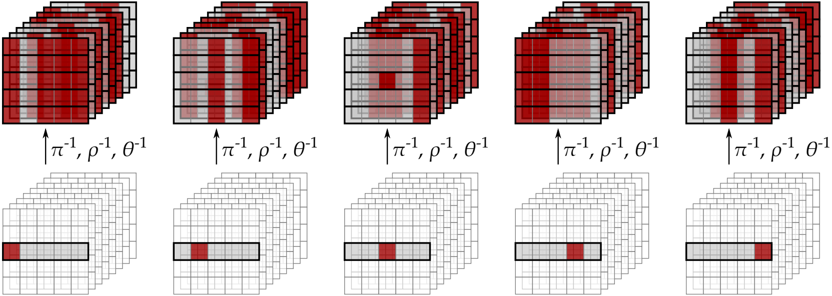
Alignment

- Property of round function
  - relative to partition of state in blocks
- **Strong alignment**
  - Low uncertainty in propagation along block boundaries
  - E.g., RIJNDAEL strongly aligned on byte boundaries
- Weak alignment
  - High uncertainty in propagation along block boundaries
  - E.g., KECCAK weakly aligned on row boundaries…

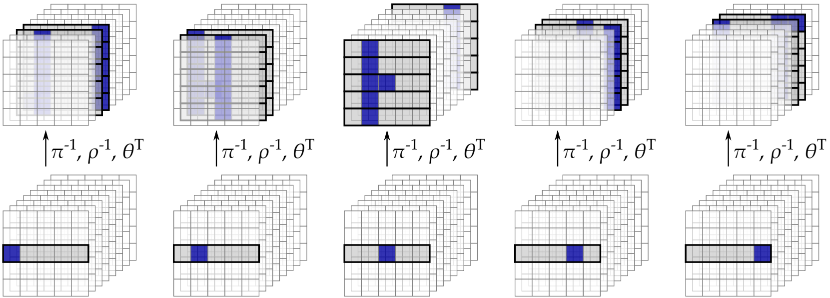
Differential patterns



Differential patterns (backwards)



Linear patterns



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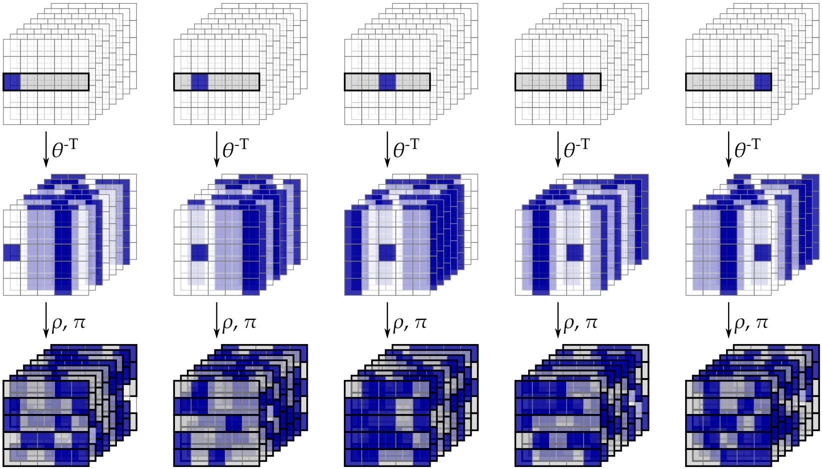


Benefits of weak alignment

Weak alignment means trails tend to diverge

- Low clustering of trails
  - Differential $b'_0 \rightarrow b'_2$, with $\mathrm{DP}(b'_0, b'_2) = \sum_{b'_1} \mathrm{DP}(b'_0, b'_1, b'_2)$
    - $b'_0 \stackrel{\lambda,\chi}{\rightarrow} b'_1 \stackrel{\lambda,\chi}{\rightarrow} b'_2$
    - $\mathrm{DP} \neq 0 \quad \Rightarrow \quad \mathrm{row}(\lambda(b'_0)) = \mathrm{row}(b'_1) \wedge \mathrm{row}(\lambda(b'_1)) = \mathrm{row}(b'_2)$
    - Weak alignment: not many b'_1 values satisfy this
- Hard to build a truncated differential trail
- Hard to mount a rebound attack
  - See also [Duc et al., Unaligned Rebound Attack: Appl. to KECCAK, FSE 2012]

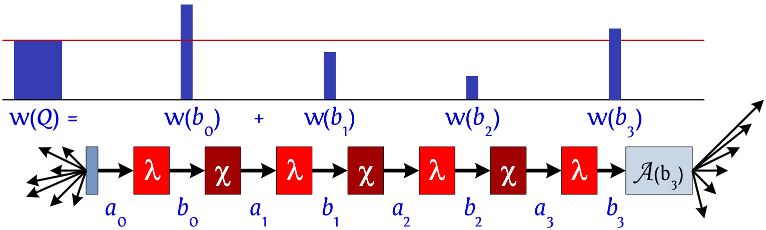
Outline

Why bound trail weights?

- Security of KECCAK relies on **absence** of exploitable trails
  …and not on presumed hardness of finding them
  $\Rightarrow$ Bound differential and linear trails as tightly as possible

How to bound trail weights?

- Bounds vs design strategies
  - ARX: no relevant bounds
  - RIJNDAEL-based: strong and simply provable bounds, **but**
    - Not for truncated differentials and rebound attack
  - Weak alignment: computer-assisted proofs are possible



- Inspired by similar efforts for
  - Noekeon [Nessie, 2000]
  - MD6 [Rivest et al., SHA-3 2008] [Heilman, Ecrypt Hash 2011]

Bounds for small instances of KECCAK

Number of rounds	Differential trails			
	$w = 1$	$w = 2$	$w = 4$	$w = 8$
2	8	8	8	8
3	16	18	19	20
4	23	29	30	46
5	30	42	≤ 54	
6	37	54	≤ 85	
16			≥ 148	
18				≥ 208

Table: Minimum weight of w-symmetric differential trails

Bounds for small instances of KECCAK

Number of rounds	Linear trails			
	$w = 1$	$w = 2$	$w = 4$	$w = 8$
2	8	8	8	8
3	16	16	20	20
4	24	30	38	46
5	30	40	≤ 66	
6	38	52	≤ 94	
16			≥ 152	
18				≥ 208

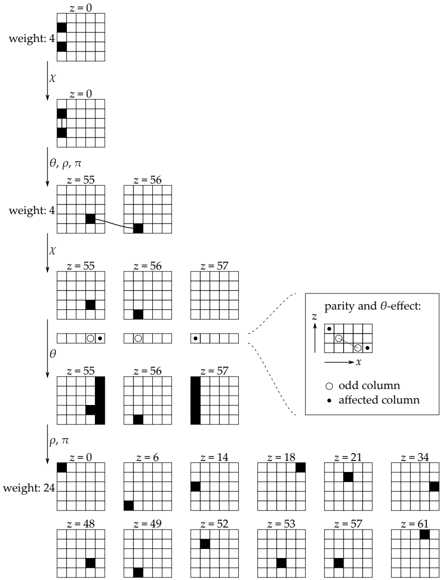
Table: Minimum weight of w-symmetric linear trails

Bounds for differential trails in KECCAK-f[1600]

Rounds	Lower bound		Best known	
1	2		2	
2	8		8	
3	32	[KECCAK team]	32	[Duc et al.]
4			134	[KECCAK team]
5			510	[Naya-Plasencia et al.]
6	74	[KECCAK team]	1360	[KECCAK team]
24	296		???	

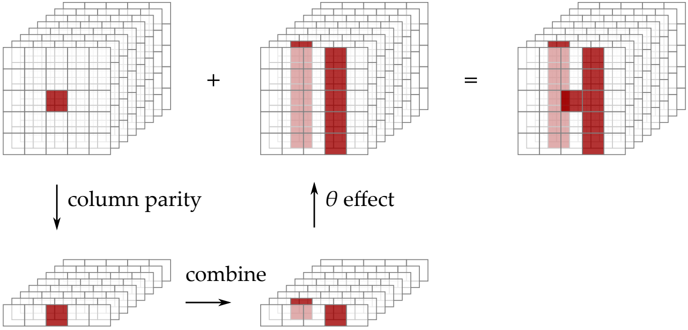
- Pessimistic view
  - Wide gap between bounds and known trails
    Open problem: narrow this gap (and also for linear trails)
  - Bound too loose to prove ideal behavior
- Optimistic view
  - Proven absence of exploitable differential trail
  - Trail weight apparently growing quickly with number of rounds

The best 3-round differential trail in KECCAK-f[1600]



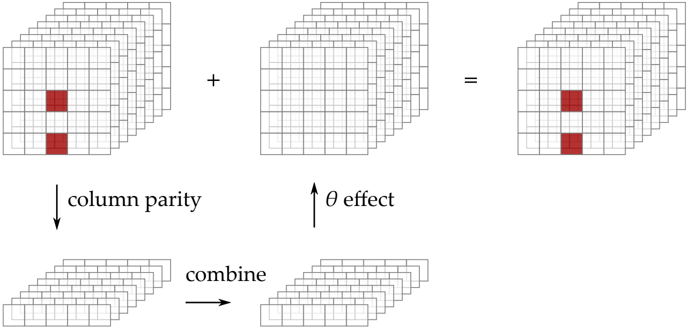
Outline

Reminder: θ, the mixing layer



- Single-bit parity flips already 10 bits
- Other linear mapping ρ and π just move bits around

Reminder: θ, the mixing layer

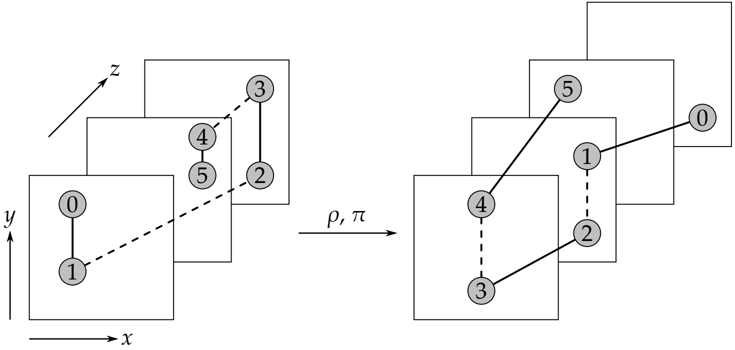


- Effect collapses if parity is zero
- The kernel

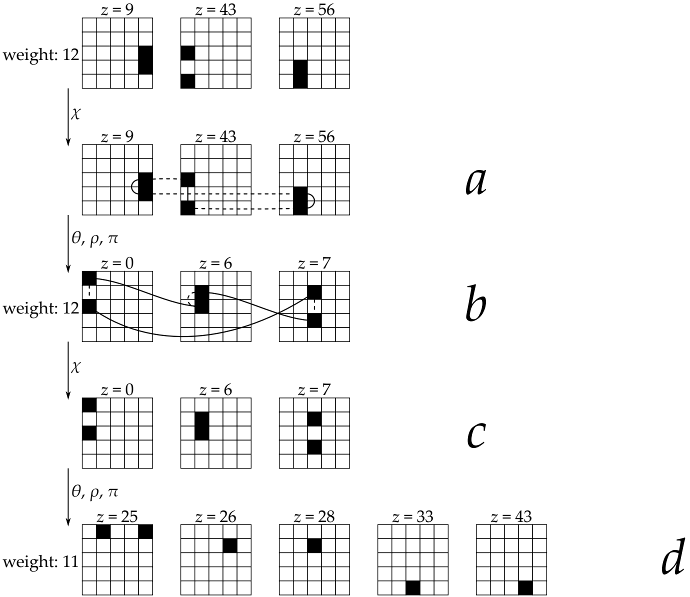
Chains

Sequence of active bits p_i with:

- p_{2i} and p_{2i+1} are in same column in a
- p_{2i+1} and p_{2i} are in same column in b



An in-kernel 3-round trail with a vortex



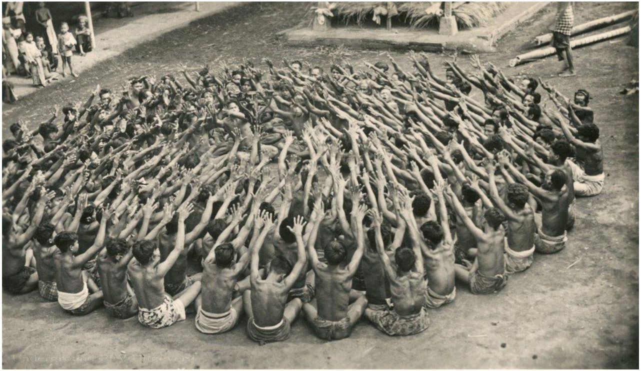
The kernel: an undesired property?

In-kernel vs non-kernel trails

- All trails (both in-kernel and non-kernel):
  - Scanned 3-round trails up to weight 36 (min. found: 32)
  - None extended to 6-round trails with weight below 74
- **In-kernel** trails:
  - Scanned 3-round trails up to weight **54** (min. found: **35**)
  - None extended to 6-round trails with weight below **82**

- Pessimistic view
  - The kernel makes θ act as the identity, clearly an undesired property
- Optimistic view
  - Staying in the kernel constrains the attacker
  - Bounds are easier to prove in the kernel

Questions?



http://sponge.noekeon.org/
http://keccak.noekeon.org/