Based on AIMA PPT slides

Artificial Intelligence 1: First-Order Logic

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Outline

- Why FOL?
- Syntax and semantics of FOL
- Using FOL
- Wumpus world in FOL
- Knowledge engineering in FOL

Pros and cons of propositional logic

☺ Propositional logic is declarative

☺ Propositional logic allows partial/disjunctive/negated information

- **(unlike most data structures and databases)**

☺ Propositional logic is compositional:

- **meaning of $B_{1,1} \wedge P_{1,2}$ is derived from meaning of $B_{1,1}$ and of $P_{1,2}$**

☺ Meaning in propositional logic is context-independent

- **(unlike natural language, where meaning depends on context)**

☹ Propositional logic has very limited expressive power

- **(unlike natural language)**
- **E.g., cannot say "pits cause breezes in adjacent squares“**
  - except by writing one sentence for each square

First-order logic

- Whereas propositional logic assumes the world contains facts,
- first-order logic (like natural language) assumes the world contains
  - **Objects: people, houses, numbers, colors, baseball games, wars, …**
  - **Relations: red, round, prime, brother of, bigger than, part of, comes between, …**
  - **Functions: father of, best friend, one more than, plus, …**

Logics in General

- Ontological Commitment: What exists in the world — TRUTH
- Epistemoligical Commitment: What an agent believes about facts — BELIEF

Language	Ontological Commitment	Epistemological Commitment
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief $\in [0,1]$
Fuzzy logic	degree of truth $\in [0,1]$	known interval value

Syntax of FOL: Basic elements

- Constants KingJohn, 2, NUS,...
- Predicates Brother, >,...
- Functions Sqrt, LeftLegOf,...
- Variables x, y, a, b,...
- Connectives $\neg, \Rightarrow, \wedge, \vee, \Leftrightarrow$
- Equality $=$
- Quantifiers $\forall, \exists$

Atomic sentences

Atomic sentence = *predicate* (*$term_1$,...,$term_n$*)
or *$term_1$ = $term_2$*

Term = *function* (*$term_1$,...,$term_n$*)
or *constant* or *variable*

- E.g., *Brother(KingJohn,RichardTheLionheart) > (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))*

Complex sentences

- Complex sentences are made from atomic sentences using connectives

$$\neg S,\ S_1 \wedge S_2,\ S_1 \vee S_2,\ S_1 \Rightarrow S_2,\ S_1 \Leftrightarrow S_2,$$

E.g. *Sibling(KingJohn,Richard)* ⇒ *Sibling(Richard,KingJohn)*

$>(1,2) \vee \leq (1,2)$

$>(1,2) \wedge \neg >(1,2)$

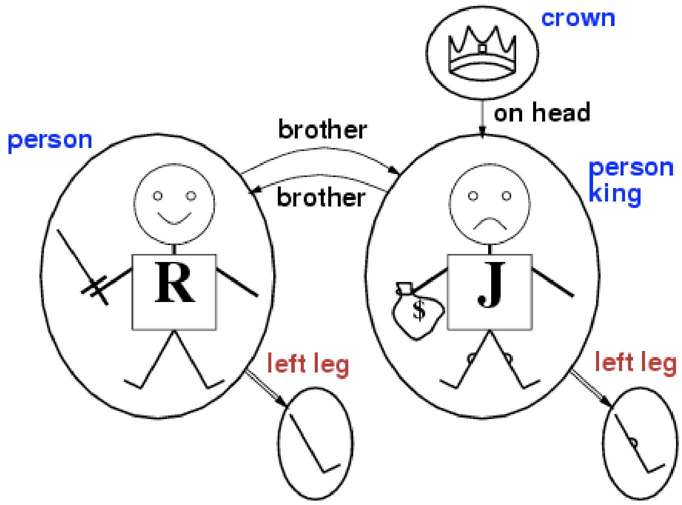
Truth in first-order logic

- Sentences are true with respect to a model and an interpretation
- Model contains objects (domain elements) and relations among them
- Interpretation specifies referents for

  constant symbols _ **objects**
  predicate symbols _ **relations**
  function symbols _ **functional relations**

- An atomic sentence *predicate*($term_1,\ldots,term_n$) is true
  iff the objects referred to by $term_1,\ldots,term_n$
  are in the relation referred to by *predicate*

Models for FOL: Example



Models for FOL

- We can enumerate the models for a given KB vocabulary:

For each number of domain elements n from 1 to ∞
  For each k-ary predicate P_k in the vocabulary
    For each possible k-ary relation on n objects
      For each constant symbol C in the vocabulary
        For each choice of referent for C from n objects . . .

- Computing entailment by enumerating the models will not be easy !!

Quantifiers

- Allows us to express properties of collections of objects instead of enumerating objects by name
- Universal: “for all” ∀
- Existential: “there exists” ∃

Universal quantification

∀<*variables*> <*sentence*>

Everyone at VUB is smart:
∀x At(x,VUB) ⇒ Smart(x)

∀x *P* is true in a model *m* iff *P* is true with *x* being each possible object in the model

Roughly speaking, equivalent to the conjunction of instantiations of *P*

At(KingJohn,VUB) ⇒ Smart(KingJohn)
∧ At(Richard,VUB) ⇒ Smart(Richard)
∧ At(VUB,VUB) ⇒ Smart(VUB)
∧ ...

A common mistake to avoid

- Typically, ⇒ is the main connective with ∀
  - **A universally quantifier is also equivalent to a set of implications over all objects**
- Common mistake: using ∧ as the main connective with ∀:

∀x At(x, VUB) ∧ Smart(x)

means "Everyone is at VUB and everyone is smart"

Existential quantification

∃<*variables*> <*sentence*>

Someone at VUB is smart:
$\exists x$ At(x, VUB) ∧ Smart(x)

$\exists x$ *P* is true in a model *m* iff *P* is true with *x* being some possible object in the model

- Roughly speaking, equivalent to the disjunction of instantiations of *P*
  At(KingJohn,VUB) ∧ **Smart(KingJohn)**
  ∨ **At(Richard,VUB)** ∧ **Smart(Richard)**
  ∨ **At(VUB, VUB)** ∧ **Smart(VUB)**
  ∨ **...**

Another common mistake to avoid

- Typically, ∧ is the main connective with ∃
- Common mistake: using ⇒ as the main connective with ∃:

  $\exists x$ At(x, VUB) ⇒ Smart(x)

  is true if there is anyone who is not at VUB!

Properties of quantifiers

∀x ∀y is the same as ∀y ∀x
∃x ∃y is the same as ∃y ∃x

∃x ∀y is not the same as ∀y ∃x
∃x ∀y Loves(x,y)

- **"There is a person who loves everyone in the world"**

∀y ∃x Loves(x,y)

- **"Everyone in the world is loved by at least one person"**

- Quantifier duality: each can be expressed using the other
  ∀x Likes(x,IceCream) ¬∃x ¬Likes(x,IceCream)
  ∃x Likes(x,Broccoli) ¬∀x ¬Likes(x,Broccoli)

Equality

- $term_1 = term_2$ is true under a given interpretation if and only if $term_1$ and $term_2$ refer to the same object
- E.g., definition of *Sibling* in terms of *Parent*:
  ∀*x,y Sibling(x,y)* ⇔ [¬(x = y) ∧ ∃m,f ¬ (m = f) ∧ Parent(m,x) ∧ Parent(f,x) ∧ Parent(m,y) ∧ Parent(f,y)]

Interacting with FOL KBs

- Suppose a wumpus-world agent is using an FOL KB and perceives a smell and a breeze (but no glitter) at *t=5*:

`Tell`**(KB,Percept([Smell,Breeze,None],5))**
`Ask`**(KB,∃a BestAction(a,5))**

I.e., does the KB entail some best action at *t=5*?

- Answer: *Yes*, {*a/Shoot*} ← substitution (binding list)
- Given a sentence *S* and a substitution α,
- *S*α denotes the result of plugging α into *S*; e.g.,

***S* = Smarter(x,y)**
α = {x/Hillary,y/Bill}
***S*α = Smarter(Hillary,Bill)**

- `Ask`(KB,S) returns some/all α such that KB |= Sα.

Using FOL

The kinship domain:

- Brothers are siblings

∀x,y *Brother(x,y)* ⇔ *Sibling(x,y)*

- One's mother is one's female parent

∀m,c *Mother(c)* = m ⇔ *(Female(m)* ∧ *Parent(m,c))*

- "Sibling" is symmetric

∀x,y *Sibling(x,y)* ⇔ *Sibling(y,x)*

- A first cousin is a child of a parent's sibling

∀x,y *FirstCousin(x,y)* ⇔ ∃p,ps Parent(p,x) ∧ Sibling(ps,p) ∧ Parent(ps,y)

Using FOL

The set domain:

- $\forall s\ Set(s) \Leftrightarrow (s = \{\}) \vee (\exists x,s_2\ Set(s_2) \wedge s = \{x|s_2\})$
- $\neg\exists x,s\ \{x|s\} = \{\}$
- $\forall x,s\ x \in s \Leftrightarrow s = \{x|s\}$
- $\forall x,s\ x \in s \Leftrightarrow [\exists y,s_2\}\ (s = \{y|s_2\} \wedge (x = y \vee x \in s_2))]$
- $\forall s_1,s_2\ s_1 \subseteq s_2 \Leftrightarrow (\forall x\ x \in s_1 \Rightarrow x \in s_2)$
- $\forall s_1,s_2\ (s_1 = s_2) \Leftrightarrow (s_1 \subseteq s_2 \wedge s_2 \subseteq s_1)$
- $\forall x,s_1,s_2\ x \in (s_1 \cap s_2) \Leftrightarrow (x \in s_1 \wedge x \in s_2)$
- $\forall x,s_1,s_2\ x \in (s_1 \cup s_2) \Leftrightarrow (x \in s_1 \vee x \in s_2)$

FOL Version of Wumpus World

- Typical percept sentence:
  Percept([Stench,Breeze,Glitter,None,None],5)
- Actions:
  Turn(Right), Turn(Left), Forward, Shoot, Grab, Release, Climb
- To determine best action, construct query:
  $\forall$ a BestAction(a,5)
- ASK solves this and returns {a/Grab}

Knowledge base for the wumpus world

- Perception
  - $\forall$b,g,t Percept([Smell,b,g],t) ⇒ Smelt(t)
  - $\forall$s,b,t Percept([s,b,Glitter],t) ⇒ Glitter(t)
- Reflex
  - $\forall$t Glitter(t) ⇒ BestAction(Grab,t)
- Reflex with internal state
  - $\forall$t Glitter(t) ∧¬Holding(Gold,t) ⇒ BestAction(Grab,t)

Holding(Gold,t) can not be observed: keep track of change.

Deducing hidden properties

$\forall$x,y,a,b *Adjacent*([x,y],[a,b]) ⇔
 [a,b] ∈ {[x+1,y], [x-1,y],[x,y+1],[x,y-1]}

Properties of locaton:

 $\forall$s,t *At*(Agent,s,t) ∧ Smelt(t) ⇒ Smelly(s)

 $\forall$s,t *At*(Agent,s,t) ∧ Breeze(t) ⇒ Breezy(s)

Squares are breezy near a pit:

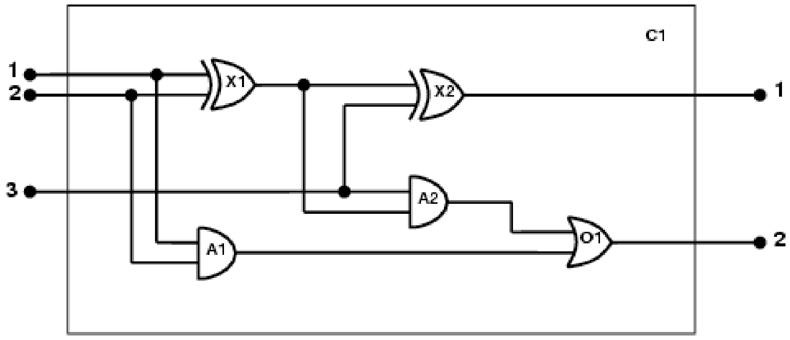
- **Diagnostic rule---infer cause from effect**
  $\forall$s Breezy(s) ⇒ ∃ r Adjacent(r,s) ∧ Pit(r)
- **Causal rule---infer effect from cause (model based reasoning)**
  $\forall$r Pit(r) ⇒ [$\forall$s Adjacent(r,s) ⇒ Breezy(s)]

Knowledge engineering in FOL

1. Identify the task (what will the KB be used for)
2. Assemble the relevant knowledge
   Knowledge acquisition.
3. Decide on a vocabulary of predicates, functions, and constants
   Translate domain-level knowledge into logic-level names.
4. Encode general knowledge about the domain
   define axioms
5. Encode a description of the specific problem instance
6. Pose queries to the inference procedure and get answers
7. Debug the knowledge base

The electronic circuits domain

One-bit full adder



The electronic circuits domain

- Identify the task
  - **Does the circuit actually add properly? (circuit verification)**
- Assemble the relevant knowledge
  - **Composed of wires and gates;**
  - **Types of gates (AND, OR, XOR, NOT)**
  - **Connections between terminals**
  - **Irrelevant: size, shape, color, cost of gates**
- Decide on a vocabulary
  - **Alternatives:**
    $Type(X_1) = XOR$
    $Type(X_1, XOR)$
    $XOR(X_1)$

The electronic circuits domain

4. Encode general knowledge of the domain
   - $\forall t_1,t_2$ **Connected**$(t_1, t_2) \Rightarrow$ **Signal**$(t_1) =$ **Signal**(t_2)
   - $\forall t$ **Signal(t) = 1** $\vee$ **Signal(t) = 0**
   - $1 \neq 0$
   - $\forall t_1,t_2$ **Connected**$(t_1, t_2) \Rightarrow$ **Connected**(t_2, t_1)
   - $\forall g$ **Type(g) = OR** $\Rightarrow$ **Signal(Out(1,g)) = 1** $\Leftrightarrow \exists n$ **Signal(In(n,g)) = 1**
   - $\forall g$ **Type(g) = AND** $\Rightarrow$ **Signal(Out(1,g)) = 0** $\Leftrightarrow \exists n$ **Signal(In(n,g)) = 0**
   - $\forall g$ **Type(g) = XOR** $\Rightarrow$ **Signal(Out(1,g)) = 1** $\Leftrightarrow$ **Signal(In(1,g))** $\neq$ **Signal(In(2,g))**
   - $\forall g$ **Type(g) = NOT** $\Rightarrow$ **Signal(Out(1,g))** $\neq$ **Signal(In(1,g))**

The electronic circuits domain

5. Encode the specific problem instance

$Type(X_1)$ = XOR **$Type(X_2)$ = XOR**

$Type(A_1)$ = AND **$Type(A_2)$ = AND**

$Type(O_1)$ = OR

Connected($Out(1,X_1),In(1,X_2)$)
Connected($Out(1,X_1),In(2,A_2)$)
Connected($Out(1,A_2),In(1,O_1)$)
Connected($Out(1,A_1),In(2,O_1)$)
Connected($Out(1,X_2),Out(1,C_1)$)
Connected($Out(1,O_1),Out(2,C_1)$)

Connected($In(1,C_1),In(1,X_1)$)
Connected($In(1,C_1),In(1,A_1)$)
Connected($In(2,C_1),In(2,X_1)$)
Connected($In(2,C_1),In(2,A_1)$)
Connected($In(3,C_1),In(2,X_2)$)
Connected($In(3,C_1),In(1,A_2)$)

The electronic circuits domain

6. Pose queries to the inference procedure

What are the possible sets of values of all the terminals for the adder circuit?

$\exists i_1,i_2,i_3,o_1,o_2$ Signal(In(1,C_1)) = $i_1 \wedge$ Signal($In(2,C_1)$) = $i_2 \wedge$ Signal($In(3,C_1)$) = $i_3 \wedge$ Signal($Out(1,C_1)$) = $o_1 \wedge$ Signal($Out(2,C_1)$) = o_2

7. Debug the knowledge base

May have omitted assertions like $1 \neq 0$

Summary

- First-order logic:
  - **objects and relations are semantic primitives**
  - **syntax: constants, functions, predicates, equality, quantifiers**
- Increased expressive power: sufficient to define wumpus world