On the use of feature selection to deal with the curse of dimensionality in microarray data

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Outline

- The ULB Machine Learning Group
- The feature selection problem
- Feature selection and bioinformatics.
- Feature selection as a stochastic optimization problem.
- A blocking idea to improve wrappers
- Final considerations.

ULB Machine Learning Group (MLG)

- 7 researchers (1 prof, 6 PhD students), 4 graduate students).
- Research topics: Local learning, Classification, Computational statistics, Data mining, Regression, Time series prediction, Sensor networks, Bioinformatics.
- Computing facilities: cluster of 16 processors, LEGO Robotics Lab.
- Website: www.ulb.ac.be/di/mlg.
- Scientific collaborations in ULB: IRIDIA (Sciences Appliquées), Physiologie Moléculaire de la Cellule (IBMM), Conformation des Macromolécules Biologiques et Bioinformatique (IBMM), CENOLI (Sciences), Microarray Unit (Hopital Jules Bordet), Service d'Anesthesie (ERASME).
- Scientific collaborations outside ULB: UCL Machine Learning Group (B), Politecnico di Milano (I), Universitá del Sannio (I), George Mason University (US).
- The MLG is part to the "Groupe de Contact FNRS" on Machine Learning.

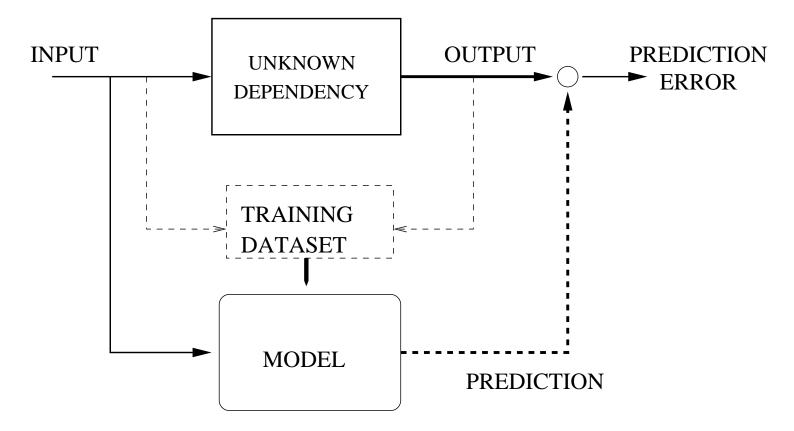
ULB-MLG: running projects

- "Integrating experimental and theoretical approaches to decipher the molecular networks of nitrogen utilisation in yeast": ARC (Action de Recherche Concertée) funded by the Communauté FranĄçaise de Belgique (2004-2009). Partners: IBMM (Gosselies and La Plaine), CENOLI.
- "COMP²SYS" (COMPutational intelligence methods for COMPlex SYStems) MARIE CURIE Early Stage Research Training funded by the European Union (2004-2008). Main contractor: IRIDIA (ULB).
- Predictive data mining techniques in anaesthesia": FIRST Europe Objectif 1 funded by the Région wallonne and the Fonds Social Européen (2004-2009). Partners: Service d'anesthesie (ERASME).
- "AIDAR Adressage et Indexation de Documents Multimédias Assistés par des techniques de Reconnaissance Vocale": funded by Région Bruxelles-Capitale (2004-2006). Partners: Voice Insight, RTBF, Titan.

Feature selection and bioinformatics

- The availability of massive amounts of experimental data based on genome-wide studies has given impetus in recent years to a large effort in developing mathematical, statistical and computational techniques to infer biological models from data.
- In many bioinformatics problems the number of features is significantly larger than the number of samples (high feature to sample ratio datasets).
- Examples can be found in the following bioinformatics tasks:
 - Breast cancer classification on the basis of microarray data.
 - Network inference on the basis of microarray data.
 - Analysis of sequence/expression correlation.

Supervised learning problems



Breast cancer classification

- Systematic investigation of dependency between expression patterns of thousands of genes (measured by DNA microarrays) and specific features of phenotypic variation in order to provide the basis for an improved taxonomy of cancer.
- It is expected that variations in gene expression patterns in different tumors could provide a "molecular portrait" of each tumor, and that the tumors could be classified into subtypes based solely on the difference of expression patterns.
- In litterature [20] classification techniques have been applied to identify a gene expression signature strongly predictive of a short interval to distant metastases in patients without tumor cells in local lymph nodes at diagnosis. In this context the number n of features equals the number of genes (ranging from 6000 to 30000) and the number N of samples is the number of patients under examinations (about hundreds).

Inference of regulatory networks

- Most biological regulatory processes involve intricate networks of interactions and it is now increasingly evident that predicting their behaviour and linking molecular and cellular structure to function are beyond the capacity of intuition.
- Dynamic regulation models are black-box prediction models which represent gene activity with continuous values. Examples of dynamic regulation models are linear relationships of the form

$$x_i(t + \delta t) = f(x_1(t), \dots, x_n(t)), \quad i = 1, \dots, n$$

where x_i is the expression level of the *i* th gene at time *t*.

 In this case, inferring a dynamic model boils down at learning the unknown dependency on the basis of expression data. These problems demand the estimation of a number of predictive models for each gene, where the number of features equals the number of measured genes.

Correlating motifs and expression levels

- These methods consists in directly correlating expression levels and regulatory motif present in presumptive transcription control regions [4, 19].
- In [4], the expression of a gene in a single experimental condition is modelled as a linear function $E = a_1S_1 + a_2S_2 + \cdots + a_nS_n$ of scores computed for sequence motifs in the upstream control region. These sequence motif scores incorporate the number of occurrences of the motifs and their positions with respect to the gene's translation start site.
- In other terms the sequence motifs of a specific gene are considered as explanatory variables (feature inputs) of a statistical model which correlates sequence features and expression of the gene. For instance the number of features is $n = 4^m$ for motifs of length m.

Feature selection

- In recent years many applications of data mining (text mining, bioinformatics, sensor networks) deal with a very large number n of features (e.g. tens or hundreds of thousands of variables) and often comparably few samples.
- In these cases, it is common practice to adopt feature selection algorithms [6] to improve the generalization accuracy.
- There are many potential benefits of feature selection:
 - facilitating data visualization and data understanding,
 - reducing the measurement and storage requirements,
 - reducing training and utilization times,
 - defying the curse of dimensionality to improve prediction performance.

The two issues of f.s.

Two main issues make the problem of feature selection a highly challenging task:

- Search in a high dimensional space: this is known to be a NP-hard problem.
- Assessment on the basis of a small set of samples: this is made difficult by the high ratio between the dimensionality of the problem and the number of measured samples.

Approaches to f.s.

Three are the main approaches to feature selection:

- **Filter methods:** they are preprocessing methods. They attempt to assess the merits of features from the data, ignoring the effects of the selected feature subset on the performance of the learning algorithm. Examples are methods that select variables by **ranking** them through compression techniques (like PCA) or by computing correlation with the output (e.g. Gram-Schmidt, mutual information).
- Wrapper methods: these methods assess subsets of variables according to their usefulness to a given predictor. The method conducts a search for a good subset using the learning algorithm itself as part of the evaluation function. The problem boils down to a problem of stochastic state space search. Example are the stepwise methods proposed in linear regression analysis.
- **Embedded methods:** they perform variable selection as part of the learning procedure and are usually specific to given learning machines. Examples are classification trees, regularization techniques (e.g. lasso).

Wrapper selection

- The wrapper search can be seen as a search in a space $S = \{0, 1\}^n$ where a generic vector $s \in S$ is such that
 - $s_j = \begin{cases} 0 & \text{if the input } j \text{ does NOT belong to the set of features} \\ 1 & \text{if the input } j \text{ belongs to the set of features} \end{cases}$
- We look for the optimal vector $s^* \in \{0,1\}^n$ such that

$$s^* = \arg\min_{s \in S} \mathsf{Miscl}(s)$$

where Miscl(s) is the generalization error of the model based on the set of variables described by s.

- the number of vectors in S is equal to 2^n .
- for moderately large n, the exhaustive search is no more possible.

Wrapping search strategies

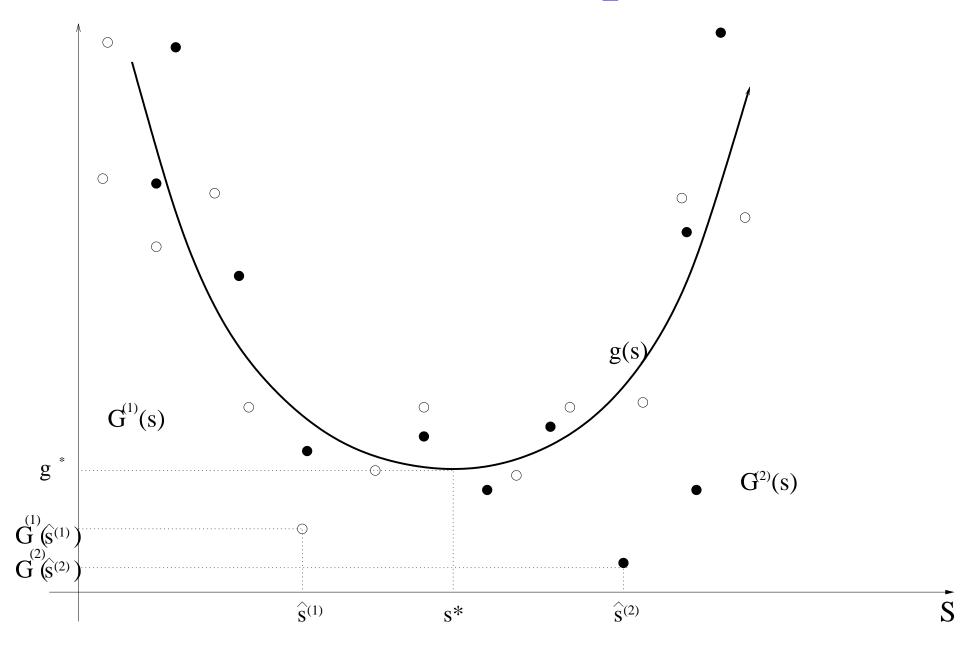
Various methods have been developed for evaluating only a small number of variables by either adding or deleting one variable at a time. We consider here some greedy strategies:

- Forward selection: the procedure starts with no variables. The first input selected is the one which allows the lowest misclassification error. The second input selected is the one that, together with the first, has the lowest error, and so on, till when no improvement is made.
- **Backward selection:** it works in the opposite direction of the forward approach. We begin with a model that contains all the *n* variables. The first input to be removed is the one that allows the lowest generalization error.
- Stepwise selection: it combines the previous two techniques, by testing for each set of variables, first the removal of features beloning to the set, then the addition of variables not in the set.

Open issues

- The wrapper approach to feature selection requires the assessment of several subset alternatives and the selection of the one which is expected to have the lowest misclassification error during generalization.
- To tackle this problem, we need to perform a search procedure in a very large space of subsets of features aiming to minimize a leave-one-out or more in general a cross-validation criterion.
- This practice can lead to a strong bias selection in the case of high dimensionality problems.
- The feature selection problem may be formulated in terms of a stochastic optimization problem where the selection of the best subset has to be based on a sample estimate Miscl.

Stochastic discrete optimization



F.s. and stochastic discrete optimization

We may formulate the feature selection problem as a discrete optimization problem [10]

$$s^* = \arg\min_{s \in S} \mathsf{Miscl}(s) \tag{1}$$

be the optimal solution of the feature selection problem and the relative optimal generalization error, respectively.

- Unfortunately the Miscl for a given s is not directly measurable but can only be estimated by the quantity $\widehat{\text{Miscl}}(s)$ which is an unbiased estimator of Miscl(s).
- The wrapper approach to feature selection aims to return the minimum \hat{s} of a cross-validation criterion $\widehat{\text{Miscl}}(s)$

$$\hat{s} = \arg\min_{s \in S} \widehat{\mathsf{Miscl}}(s)$$
 (2)

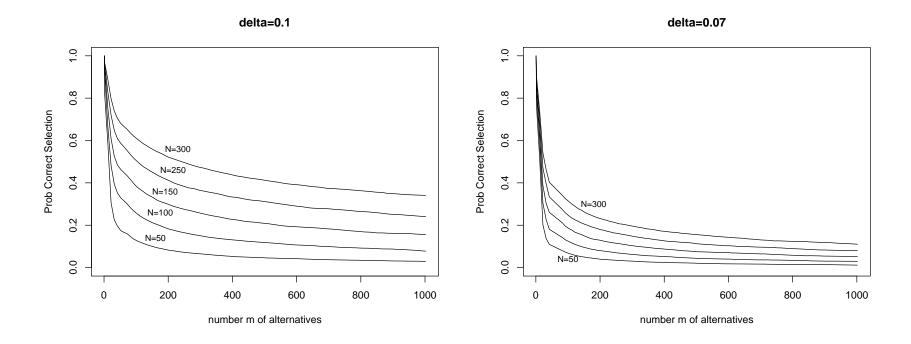
Probability of correct selection

Let $Miscl(s_j)$, j = 1, ..., m be the (unknown) misclassification error of the *j*th feature subset and assume that the subsets are indexed in such a way that $Miscl(s_m) \leq Miscl(s_{m-1}) \leq \cdots \leq Miscl(s_1)$ so that (unknown to us) the feature set $s_m = s^*$ is the best one. Assume also that $Miscl(s_{m-1}) - Miscl(s_m) \geq \delta > 0$.

$$\begin{split} \operatorname{Prob}\left\{ \hat{\mathbf{s}} = s^* \right\} &= \operatorname{Prob}\left\{ \widehat{\operatorname{Miscl}}_m < \widehat{\operatorname{Miscl}}_j, \forall j \neq m \right\} \geq \\ &\geq \operatorname{Prob}\left\{ \mathbf{Z}_j < \frac{\delta}{\sigma_m \sqrt{4/N}}, \forall j \neq m \right\} \end{split}$$

where $\text{Miscl}_j - \text{Miscl}_m \ge \delta > 0$ for all j = 1, ..., m - 1 and the random vector $(\mathbf{Z}_1, \mathbf{Z}_2, ..., \mathbf{Z}_{m-1})$ has a multivariate Normal distribution with means 0, variances 1 and common pairwise correlations 1/2 [9].

Lower bounds



Lower bound on the probability of correct selection vs. the number of alternative subsets for $N = \{50, 100, 150, 200, 250\}$

Our proposal

- We propose an original *blocking* strategy for improving feature selection which aggregates in a paired way the validation outcomes of several learning algorithms to assess a gene subset and compare it to others.
- This is new with respect to conventional wrappers which commonly adopt only one learning algorithm to evaluate the relevance of a given set of variables.
- The rationale of the approach is that by increasing the amount of experimental conditions under which we validate a feature subset, we can lessen the problems related to the scarcity of samples and consequently come up with a better selection.
- The idea is quite simple: if we want to search for the best subset of features, this subset should appear among the best ones whatever the learning algorithm is. So far, the methods in feature selection end up with one of these two ways: either avoid any learning algorithm (as in the case of filters), either restrict themselves to consider only a specific one (as in conventional wrapper). What we advocate here is that by using more than one algorithm in a feature selection task we may improve the robustness of the selection procedure.

Datasets

Name	Reference	Platform	N	n	C
Golub	[5]	Affy hu6800	72	7129	2
SRBCT	[8]	cDNA	63	2308	4
ALL	[22]	Affy hgu95av2	248	12558	6
Hedenfalk	[7]	cDNA	22	3226	3
Alon	[1]	Affy hum6000	62	2000	2
Notterman	[11]	Affy hu6800	36	7457	2
West	[21]	Affy hu6800	49	7129	4
9Tumors	[17]	Affy hu6800	60	7129	9
11Tumors	[18]	Affy hgu95av2	174	12533	11
14Tumors	[14]	Affy hu35ksubacdf	308	15009	26
LungCancer	[3]	Affy hgu95av2	203	12600	5
BrainTumor1	[13]	Affy hu6800	60	7129	5
BrainTumor2	[12]	Affy hgu95av2	50	12625	4
DLBCL	[15]	Affy hu6800	77	7129	2
Leukemia2	[2]	Affy hgu95a	72	12582	3
Prostate	[16]	Affy hgu95av2	102	12600	2

Experimental session

- A first dimensionality reduction step is carried out by ranking the variables according to an univariate classification and by selecting the first 1000 gene probes.
- A three-fold cross-validation strategy is used to measure the generalization accuracy of the two competitive feature selection strategies. Note that unlike conventional cross-validation we use here only one third of the samples for training and two third of the samples for test.
- Given a third of the samples, we run both a conventional forward selection with a given learning algorithm and an enhanced forward selection based on K = 6 learning algorithms implemented by the R statistical software. The six algorithms are: a Naive Bayes (NB), a Support Vector Machine with a radial Gaussian kernel (SVM), a Support Vector Machine with a linear kernel (SVML), a Nearest Neighbour with one neighbor (NN1), a Nearest Neighbor with 5 neighbors (NN5) and a Nearest Neighbor with 10 neighbors (NN10).

Results

Name	NB	NB*	SVM	SVM*	SVML	SVML*	NN1	NN1*	NN5	NN5*	NN10	NN10*
Golub	6.94	5.6	6.9	5.6	11.1	5.6	12.5	5.6	18.1	8.3	16.7	16.7
SRBCT	39.7	20.6	42.9	23.8	22.2	15.9	42.9	20.6	36.5	33.3	63.5	52.4
ALL	11.7	9.3	16.1	9.3	13.7	10.9	14.5	12.1	11.3	9.3	13.7	9.7
Hedenfalk	40.9	45.5	68.2	45.5	40.9	40.9	50	45.5	45.5	27.3	59.1	50
Alon	22.6	19.4	22.6	21	22.6	17.7	19.4	21	17.7	11.3	17.7	12.9
Notterman	13.9	25	11.1	16.7	8.3	16.7	22.2	16.7	11.1	16.7	27.8	19.4
West	42.9	44.9	42.9	40.8	49	42.9	61.2	42.9	49	42.9	57.1	53.1
9Tumors	91.7	75	76.7	75	81.7	76.7	80	73.3	81.7	70	81.7	80
11Tumors	46	39.1	37.9	39.7	31	37.9	39.7	35.1	56.9	37.9	48.9	43.7
14Tumors	68.8	72.4	67.9	71.8	72.7	65.3	67.9	66.2	69.5	69.2	73.4	70.8
LungCancer	17.2	13.3	13.8	11.3	14.3	11.8	17.7	10.8	11.3	12.3	18.7	16.7
BrainTumor1	31.7	31.7	35	33.3	38.3	41.7	41.7	31.7	26.7	35	40	31.7
BrainTumor2	26	18	30	20	22	22	28	16	36	20	32	20
DLBCL	26	20.8	27.3	26	20.8	23.4	29.9	24.7	10.4	23.4	24.7	24.7
Leukemia2	12.5	9.7	19.4	9.7	13.9	4.2	12.5	8.3	16.7	9.7	12.5	8.3
Prostate	14.7	10.8	10.8	13.7	10.8	14.7	9.8	12.7	13.7	17.6	10.8	12.7
AVG	34.1	31.2	33.6	31.4	32.1	30	34.7	29.7	34.1	30.8	37.7	34

For a given dataset and algorithm, the lowest figure is reported in bold notation if the misclassification accuracy of the corresponding selection method is significantly lower.

Some considerations

- Bioinformatics applications are known to be characterized by highly noisy data.
- The huge size of the feature space compared to the number of samples makes hard the problem in terms of:

Optimization techniques to explore the feature space.

Large variance of the resulting model.

- Biologists asks for prediction accuracy but mainly for causual intepretation (gene signature).
- Biologists are scared of unstable feature selection procedures which change the sets of relevant genes simply by adding more observations. Examples are clinical study with different populations of patients.
- Filtering techniques are computational efficient and robust against overfitting.
 They may introduce bias but may have considerably less variance.
- Literature have been inflationed by over-optimistic results.

Closing remarks

- Let us not forget that any learner is an estimator and as such any outcome it returns, is a random variable.
- If the outcome of our feature selection technique is a set of variables, this set is also a random set.
- Data miners are used to return confidence interval on accuracy.
- They should start returning confidence intervals also on feature (gene) subsets.

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