The Effect of Infinity on Matrix-Analytic Models

Project description and selection criteria

Project description

Recent results have shown that the behaviour and properties of Quasi-Birth-and-Death (QBD) processes with infinitely many phases can be starkly different to those of the traditional QBDs with finitely many phases. The main theme of the ARC project is to study in greater depth these differences and to extend the analysis to related processes such as fluid queues.

Three general research projects form the basis of the work that will be undertaken by the two PhD students and the postdoctoral fellow respectively. These projects bring together related themes and are coherent in respect of the specific background that they require.

A. Decay Rates in Stochastic Fluid Models This project is about the further development of the theory of stochastic fluid queues, which currently deals for the most part with systems of finitely many phases. While they are related to QBDs and to processes of M/G/1 and GI/M/1 type, fluid queues are studied in a slightly different way, which leads to different equations for the fundamental matrices involved. The principal issue that needs to be resolved is to find a way to characterize the decay rate of a stochastic fluid model independently of the matrix K of Ramaswami's formulation. This will be the first task in this project, after which specific questions will be addressed. On the one hand, one should pursue the analysis of fluid queues with as general a phase-transition structure as possible, on the other hand we might examine some specific cases found in the literature, the first of which being the fluid queue fed by an M/M/1 queue.

B. Decay Rates in Discrete-Level Matrix-Analytic Models As opposed to fluid models, the level in QBDs and processes of M/G/1 and GI/M/1 type is discrete. This project amalgamates different themes of our grant application in which we propose to employ 'traditional' matrix analytic techniques to study questions about decay rates: the effect of the transition structure at the boundary on asymptotic properties of the stationary queue length distribution, convergence of finite truncations to the untruncated system, conditions for non-geometric decay, . . . The classes of equations that are typically produced in the study of discrete-level models are fundamentally the same, and are thus amenable to study as part of a single project. The questions posed above have been studied in the literature under various, sometimes restrictive, assumptions. The aim of this second project is to gain a comprehensive and general understanding of the whole area by reviewing and extending this work.

C. Relationship Between a Large Deviations Approach and a Matrix-Analytic approach to Decay Rates This project follows a theme of the original application and which was appreciated by a couple of the assessors. Part of the background knowledge required for this study comes from outside the field of matrix-analytic methods and thus lies sensibly in a separate project. The initial approach will be to attempt to construct a large deviations principle for the probability that a QBD reaches a high level and to relate the equations that come out of such a formulation with those that arise in a traditional matrix-analytic approach.

While we plan to retain the flexibility to allocate projects to the PhD students and the the postdoctoral fellow in the most appropriate way, depending on their individual backgrounds and interests, it is likely that the first two projects will be more suitable for PhD study and the third for the postdoctoral fellow to work on.

Selection criteria — Postdoctoral position

The essential minimum criteria are:

1. A PhD or equivalent in an area of applied probability, computational mathematics, statistics, computer science or a related field. Candidates who are nearing completion of their PhD are also acceptable. Note, however, that such candidates would be expected to work full-time on the project.
2. An interest in pursuing research on matrix-analytic methods in applied probability.
3. Ability to communicate clearly and effectively, both verbally and in writing.
4. Ability to work as part of a team.
5. The flexibility to spend some time visiting Professor Taylor's group at the University of Melbourne.

Desirable characteristics are:

6. Demonstrated record of research achievements, as evidenced by journal publications and conference papers.
7. A knowledge of matrix-analytic methods in applied probability and/or large deviations theory would be well-regarded.
8. An interest in the application of stochastic modeling to physical problems.
9. A willingness to become involved in high-level tutorial and supervisory activities.

Selection criteria — Doctoral fellows

The essential minimum criteria are:

1. A license or masters degree or equivalent in an area of mathematics, statistics, computer science or a related field. Candidates who are nearing completion of their degree may also apply. Note, however, that such candidates would be expected to complete their degree before commencing in the position.
2. An interest in pursuing research on matrix-analytic methods in applied probability.
3. Ability to communicate clearly and effectively, both verbally and in writing.
4. Ability to work as part of a team.
5. The flexibility to spend some time visiting Professor Taylor's group at the University of Melbourne.

Desirable characteristics are:

6. Demonstrated record of scholarly excellence.
7. A knowledge of applied probability or stochastic modeling.
8. An interest in the application of stochastic modeling to physical problems.
9. A willingness to become involved in tutorial and supervisory activities.