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The Monte Carlo method in Excel : a Primer

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This note gives a brief introduction to the Monte Carlo method (a technique to generate samples of random variables) using Excel. The method appeared in 1949 with the first computer – the ENIAC (see Metropolis (1987), for the story of the method by one of his fathers).

The idea is the following. You set up an Excel worksheet to analyze a problem. One or several variables are random variables. You want to randomly simulate the problem solution. To do this, you generate values for the random variable or, to use statistical language, you generate samples.

Let us start with an example. Consider a 5-year project with an initial cost of €1,000. Cash flows grow at rate g. The cash flow in year t is $C_t = C_{t-1}(1+g)$. The net present value (NPV) of the project is:

$$NPV = -1,000 + \frac{C_1}{1+r} + \frac{C_1(1+g)}{(1+r)^2} + ... + \frac{C_1(1+g)^4}{(1+r)^5}$$

where r is the discount rate.

You prepare the following Excel spreadsheet.

	A	B	C	D	E	F	G
1	Initial investment	1,000					
2	Cash flow year 1 C_1	300					
3	Growth rate g	5%					
4	Discount rate r	10%					
5							
6	Year	0	1	2	3	4	5
7	g			5%	5%	5%	5%
8	Cash flow	-1,000.00	300.00	315.00	330.75	347.29	364.65
9	Discount factor	1.000	0.909	0.826	0.751	0.683	0.621
10	NPV	245.18					

The project looks profitable, the NPV is positive. But you realize that the growth rate in your calculation is an expected value. You want to do some sensitivity analysis to understand how sensitive the NPV is to changes in the growth rate.

You could, of course, simply change the value of g in the spreadsheet and record the result. But for a large number of values, this would be cumbersome. Luckily Excel has a very useful command, the Data|Table command, to do sensitivity analysis.

To create a Data|Table, proceed as follow.

1. put the values of the variable (the growth rate in our example) in a range of cells;
2. put the value of the result that you wish to record at the top of the adjacent column.

Your spreadsheet now looks like this:

	A	B	C	D	E	F	G
1	Initial investment	1,000					
2	Cash flow year 1 C_1	300					
3	Growth rate *g*	5%					
4	Discount rate *r*	10%					
5							
6	Year	0	1	2	3	4	5
7	g			5%	5%	5%	5%
8	Cash flow	-1,000.00	300.00	315.00	330.75	347.29	364.65
9	Discount factor	1.000	0.909	0.826	0.751	0.683	0.621
10	NPV	245.18					
11							
12	Sensitivity analysis	*g*	245.18	C12. =B10			
13		-5%					
14		0%					
15		5%					
16		10%					
17		15%					

Now, to create the data table, proceed as follow:

1. Select the Data|Table range (B12:C17 in our example);
2. Choose Table in the Data menu;
3. Type B3 (the cell with the value of *g*) in Column Input Cell box.

Here is the result.

	A	B	C	D	E	F	G
1	Initial investment	1,000					
2	Cash flow year 1 C_1	300					
3	Growth rate *g*	5%					
4	Discount rate *r*	10%					
5							
6	Year	0	1	2	3	4	5
7	g			5%	5%	5%	5%
8	Cash flow	-1,000.00	300.00	315.00	330.75	347.29	364.65
9	Discount factor	1.000	0.909	0.826	0.751	0.683	0.621
10	NPV	245.18					
11							
12	Sensitivity analysis	*g*	245.18	C12. =B10			
13		-5%	39.09				
14		0%	137.24				
15		5%	245.18				
16		10%	363.64				
17		15%	493.37				

As you can see, Excel provides, next to each growth rate, the corresponding NPV.

You now want to go one step further. The growth rate might vary from one year to another. Let us suppose that *g* is a normally distributed random variable with a 5% mean and a 10% standard deviation *g*~N(5%, 10%). You would like to know the probability distribution of the NPV under this assumption. Using the Monte Carlo method is the solution to your problem.

The idea is pretty simple. You let your computer draw randomly values for the annual growth rates and calculate the corresponding NPV. Here is an example:

	A	B	C	D	E	F	G
1	Initial investment	1,000					
2	Cash flow year 1 C_1	300					
3	Growth rate *g*	5%	StDev	10%			
4	Discount rate *r*	10%					
5							
6	Year	0	1	2	3	4	5
7	g			-9.39%	7.23%	0.46%	-5.17%
8	Cash flow	-1,000.00	300.00	271.84	291.50	292.85	277.72
9	Discount factor	1.0000	0.9091	0.8264	0.7513	0.6830	0.6209
10	NPV	88.86					

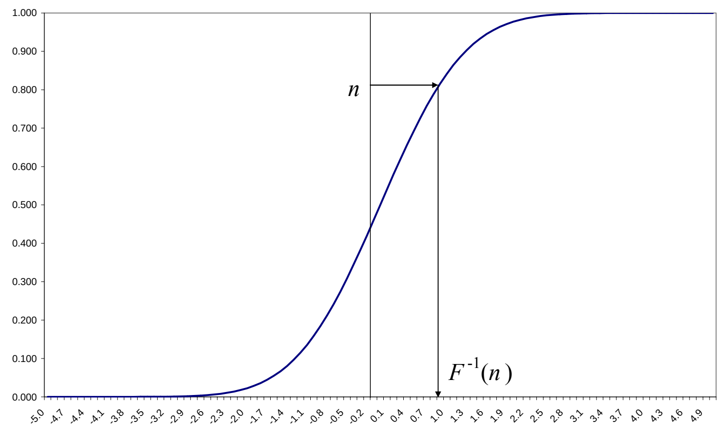
Let us is in more detail how this example was generated. For that purpose, we should first understand random numbers.
The core of the Monte Carlo method is a random-number generator. A random number is a uniformly distributed random variable over the interval [0,1]. Random numbers can be generated using the Excel Rand() function. This function produces a new random number every time the spreadsheet is recalculated (to recalculate, hit the F9 button).

Here is an example:

	A	B	C	D
1	Generating random numbers			
2				
3	Random number =	0.566259	B3. =RAND()	
4				

The next step is to transform uniformly distributed random numbers into a non uniform distribution. In this note, we will limit the presentation to normally distributed random variables.

Consider a standard normal variable $Z\sim N(0,1)$; Let $F(z)$ be the cumulative normal probability distribution: $F(z)$ = Proba($Z \leq z$). Pick a number $0<n<1$. The value of z such that $F(z) = n$ is
$z = F^{-1}(n)$
where F^{-1} is the inverse of the cumulative normal probability function



Remember the formula for $F(z)$.

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{x^2}{2}} dx$$

This formula is obviously not user friendly to manipulate. But Excel comes to the rescue with the function NORMSINV() that gives the value of this inverse function.

To generate values of the standard normal variable, use a random number for *n*. This is easily done in Excel with the formula NORMSINV(RAND()).

If the underlying random variable X is $N(\mu,\sigma)$, values are calculated by $X = \mu+\sigma Z$.

As an illustration, the formula used to calculate the growth rate in our example is
=$B$3+$D$3*NORMSINV(RAND())
Where $B$3 is the average growth rate and $D$3 is the standard deviation.

The final step is to recalculate the results many times. To do this, use the Data Table command which let you recalculate the spreadsheet by changing one variable. The trick is simply to choose an empty cell as the value to be changed.

The following table illustrates the method for our example with 10 simulations.

	A	B	C	D	E	F	G
1	Initial investment	1,000					
2	Cash flow year 1 C_1	300					
3	Growth rate *g*	5%	StDev	10%			
4	Discount rate *r*	10%					
5							
6	Year	0	1	2	3	4	5
7	g			11.66%	-7.68%	17.62%	-1.04%
8	Cash flow	-1,000.00	300.00	334.99	309.27	363.75	359.96
9	Discount factor	1.0000	0.9091	0.8264	0.7513	0.6830	0.6209
10	NPV	253.89					
11							
12	Simulation	NPV					
13		253.89		Mean	247.98		
14	1	305.53		StDev	168.92		
15	2	92.72		Min	63.82		
16	3	532.51		Max	532.51		
17	4	463.27		5% Percentile	66.77		
18	5	163.94					
19	6	172.72					
20	7	399.17					
21	8	70.38					
22	9	215.78					
23	10	63.82					

The number of simulation in the previous table has been chosen to illustrate the procedure. In a real application, the number should be greater in order for the estimated value of the mean of the NPV to be sufficiently precise.

This is illustrated in the following table which compares the results obtained when recalculating 10 times the spreadsheet for 10 simulations and 1,000 simulations.

	10 simulations									
	1	2	3	4	5	6	7	8	9	10
Mean	194.22	253.55	198.89	210.65	229.19	235.79	283.61	243.62	239.87	254.67
StDev	115.73	106.36	162.51	95.68	117.10	121.68	117.38	135.01	131.03	138.09
Min	-36.76	114.17	-101.32	145.69	35.91	50.60	121.25	45.78	82.85	-14.90
Max	342.50	388.76	464.93	409.90	396.35	420.23	441.25	443.67	531.81	463.64
5% Percentile	26.37	117.62	-36.83	146.26	61.55	67.65	153.79	61.29	90.69	55.23
	1,000 simulations									
Mean	244.40	244.48	249.81	249.47	238.26	246.53	244.18	238.68	250.91	242.80
StDev	127.21	116.26	126.30	122.73	129.96	124.78	127.00	126.79	121.01	123.75
Min	-87.37	-66.51	-154.14	-94.44	-95.74	-97.49	-165.09	-161.38	-113.91	-144.87
Max	672.89	692.35	669.65	709.11	739.97	667.53	788.87	702.57	627.51	719.24
5% Percentile	35.79	50.19	47.87	57.58	33.57	48.88	49.60	46.21	57.77	51.11

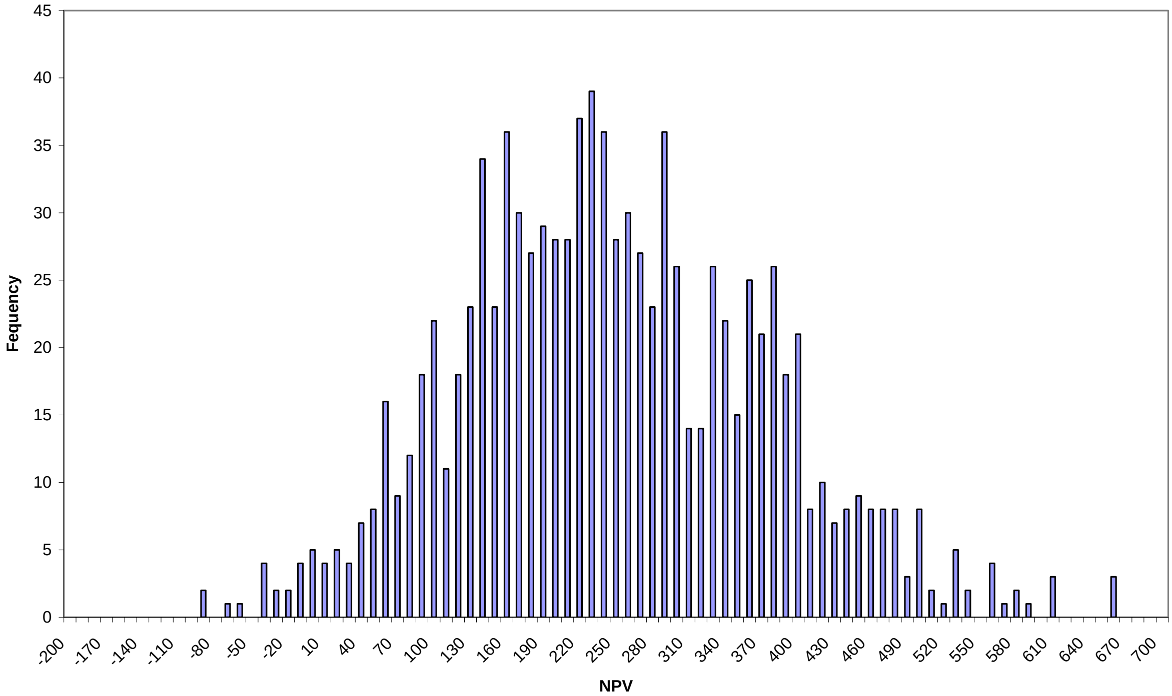
As you can see, the summary statistics vary quite a lot when recalculating with time simulations but are more stable with 1,000 simulation.

The degree of precision of the estimated mean is given by the standard error

$$SE = \frac{\hat{\sigma}}{\sqrt{N}}$$

Using the first sample, SE = 36.60 with 10 simulation and drops to 4.02 with 1,000 simulations.

The next figure illustrate the frequency distribution of the NPV for 1,000 simulations.



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