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## Interest Rate Futures

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Interest rates futures (IRF) are among the oldest and most popular financial futures contracts. The first contract, the Eurodollar futures, was created in 1975, by the Chicago Mercantile Exchange. Since then, similar contracts have been introduced on all main derivative markets.

An IRF is similar to a FRA. Like a FRA, the payoff at maturity is the difference between a fixed interest rate interest payment and a floating interest rate payment. The interest period is 3 months. The buyer of an IRF receives the fixed rate and pays the floating rate. This is the opposite of an FRA: buying an IRF is similar to selling an FRA.

Let us start with an example : the Three Month Euro Interest Rate Futures contract traded on Liffe. The underlying interest rate is the 3 month Euribor rate. The size of one contract (a notional amount used to calculate interest) is €1,000,000. On October 15, 2003, the quotation for the March 2004 was 97.75. This means that (ignoring marking to market) the buyer of a futures contract will received end of March an amount equal to the interest on €1,000,000 for 3 months based on a interest rate equal to  $(100 - 97.75)\% = 2.25\%$ . He will have to pay an amount equal to the interest on the notional amount for 3 month based on the 3m Euribor rate prevailing at maturity.

The way the quotation of the IRF is expressed (100 – interest rate) makes it similar to other futures contract. The buyer gains if the futures prices increases; he or she makes a loss if the futures price drops. The gain or the loss can be calculated quite simply by multiplying the change in the futures price (expressed in basis point) by a tick equal to the value of 1 basis point. If the size of the contract is 1 million units of currency, the value of the tick is 25 units of currency:

$$\text{Tick} = \text{Size of contract} \times (0,01/100) \times 3/12$$

IRF contracts are designed so that the quoted price increases when interest rates go down and decreases when interest rates go up. A borrower would, as a consequence, hedge his or her position by selling an IRF. An investor would buy an IRF.

The quoted price of an IRF is expressed as 100 minus a 3-month interest rate. Notice that this does not correspond to a price. If  $F_t$  is the quoted price at time  $t$ , the underlying interest rate is:

$$R_t = (100 - F_t)/100$$

As an example, a quote of 97.75 correspond to an interest rate of 2.25%

The quoted price at maturity is set equal to 100 minus the underlying interest rate (in%) :

$$F_T = 100 - r_T \times 100$$

If the 3-month Euribor is 3% at maturity, the quote would be 97.

The payoff on a long position is calculated by multiplying the change in the quoted futures price (in bp) by the tick :

$$\text{Payoff at maturity} = \text{Tick} \times (F_T - F_t) \times 100$$

Replacing  $F_T$ ,  $F_t$  and Tick by their expression leads to :

$$\text{Payoff at maturity} = \text{Size of contract} \times \frac{0.01}{100} \times \frac{3}{12} \times [(100 - 100r_T) - (100 - 100R_t)] \times 100$$

This expression simplifies to :

$$\text{Payoff at maturity} = \text{Size of contract} \times (R_t - r_T) \times 3/12$$

This last expression shows that the buyer of an IRF implicitly receives the fixed rate and pays the floating rate.

Example : Suppose you buy the Liffe March 04 IRF futures contract at 97.75. You lock in a 3-month rate equal to  $100 - 97.75 = 2.25$  percent. If the 3-month Euribor rate at maturity is 3%, the payoff is:

$$\text{Payoff at maturity} = 1.000.000 \times (2,25\% - 3,00\%) \times 3/12 = -1,875 \text{ €}$$

The same result can be obtained using the tick. The futures price at maturity is  $100 - 3 = 97$ . The change in the futures price is  $97 - 97.75 = -0.75$  (= -75 basis points). The payoff is:

$$(-75) \times 25 \text{ €} = -1,875 \text{ €}$$

We mentioned that a long IRF is *similar* to a short FRA. There is a subtle difference. Both contract involve the difference between a fixed rate and a floating rate. In the case of a FRA, the payment at the maturity is discounted from the end of the interest period whereas this is not the case for an IRF.

Let  $C_T$  be the payoff at maturity :

$$C_{T,\text{IRF}} = M \times (R_t - r_T) \times 3/12$$

$$C_{T,\text{fra}} = \frac{M \times (r_T - R_t) \times 3/12}{1 + r_T \times 3/12}$$

So:

$$C_{T,\text{IRF}} = - C_{T,\text{fra}} \times a$$

$$\text{with } a = 1/(1 + r_T \times 3/12)$$

As  $a$  is less than 1, the value of an IRF is slightly below the value of a FRA.