

Derivatives Introduction

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- **Reference:**

John HULL *Options, Futures and Other Derivatives*, Seventh edition,
Pearson Prentice Hall 2008

John HULL *Options, Futures and Other Derivatives*, Sixth edition,
Pearson Prentice Hall 2006

Probably the best reference in this field. Widely used by practioners.

- Copies of my slides will be available on my website:

www.ulb.ac.be/cours/solvay/farber

- **Grades:**

- Cases: 30%
- Final exam: 70%

New: Stéphanie Collet
Tutorial: Tuesday 12am – 2pm
UA4-222

Course outline

1. Introduction
2. Pricing a forward/futures contract
3. Hedging with futures
4. IR derivatives
5. IR and currency Swaps
6. Introduction to option pricing
7. Inside Black-Scholes
8. Greeks and strategies
9. Options on bonds and interest rates (1)
10. Options on bonds and interest rates (2)
11. Credit derivatives (1)
12. Credit derivatives (2)

5 ECTS = 125 hours!!!

- Classes 24 h (12 × 2h)
- Reading 24 h (12 × 2h)
- Review cases 12 h (12 × 1h)
- Graded cases 32 h (2 × 16h)
- Prep exam 40 h (5 × 8h)
- Exam 3 h
- Total 135 h

1. Today

1. Course organization
2. Derivatives: definition (forward/futures), options + brief history
3. Derivatives markets: evolution + BIS statistics
4. Why use derivatives
5. Forward contracts: cash flows + credit risk
6. Futures: marking to market, clearing house
7. Valuing a forward contract: key idea (no arbitrage)

Additional references

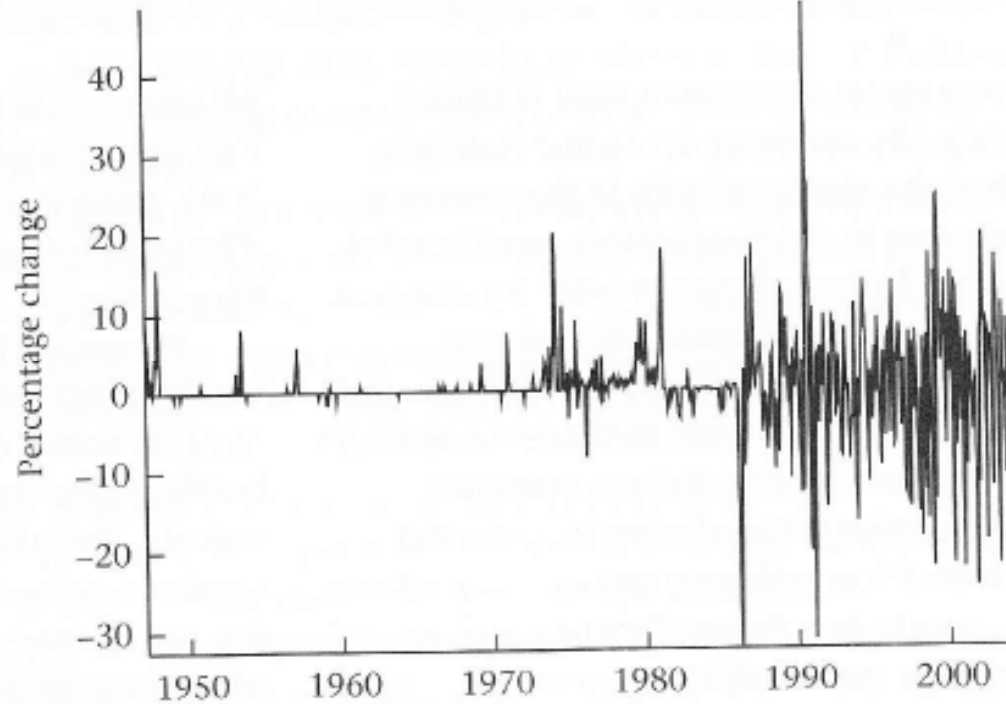
- Chance, Don, *Analysis of Derivatives for the CFA Program*, AIMR 2003
- Cox, John and Mark Rubinstein, *Options Markets*, Prentice-Hall 1985
- Duffie, Darrell, *Futures Markets*, Prentice Hall 1989
- Hull, John, *Risk Management and Financial Institutions*, Pearson Education 2007
- Jarrow, Robert and Stewart Turnbull, *Derivative Securities*, South-Western College Publishing 1994
- Jorion, Philippe, *Financial Risk Manager Handbook*, 2d edition, Wiley Finance 2003
- McDonal, Robert, *Derivatives Markets*, 2d edition, Addison Wesley 2006
- Neftci, Salih, *An Introduction to the Mathematics of Financial Derivatives*, 2d ed., Academic Press 2000
- Neftci, Salih, *Principles of Financial Engineering*, Elsevier Academic Press 2004
- Portait, Roland et Patrice Poncet, *Finance de marché: instruments de base, produits dérivés, portefeuilles et risques*, Dalloz 2008
- Siegel, Daniel and Diane Siegel, *The Futures Markets*, McGraw-Hill 1990
- Stulz, René, *Risk Management and Derivatives*, South-Western Thomson 2003
- Wilmott, Paul, *Derivatives: The Theory of Practice of Financial Engineering*, John Wiley 1998

- A derivative is an instrument whose value depends on the value of other more basic underlying variables
- 2 main families:
 - Forward, Futures, Swaps
 - Options
- = DERIVATIVE INSTRUMENTS
 - value depends on some underlying asset

Derivatives and price variability: oil

FIGURE 1.1

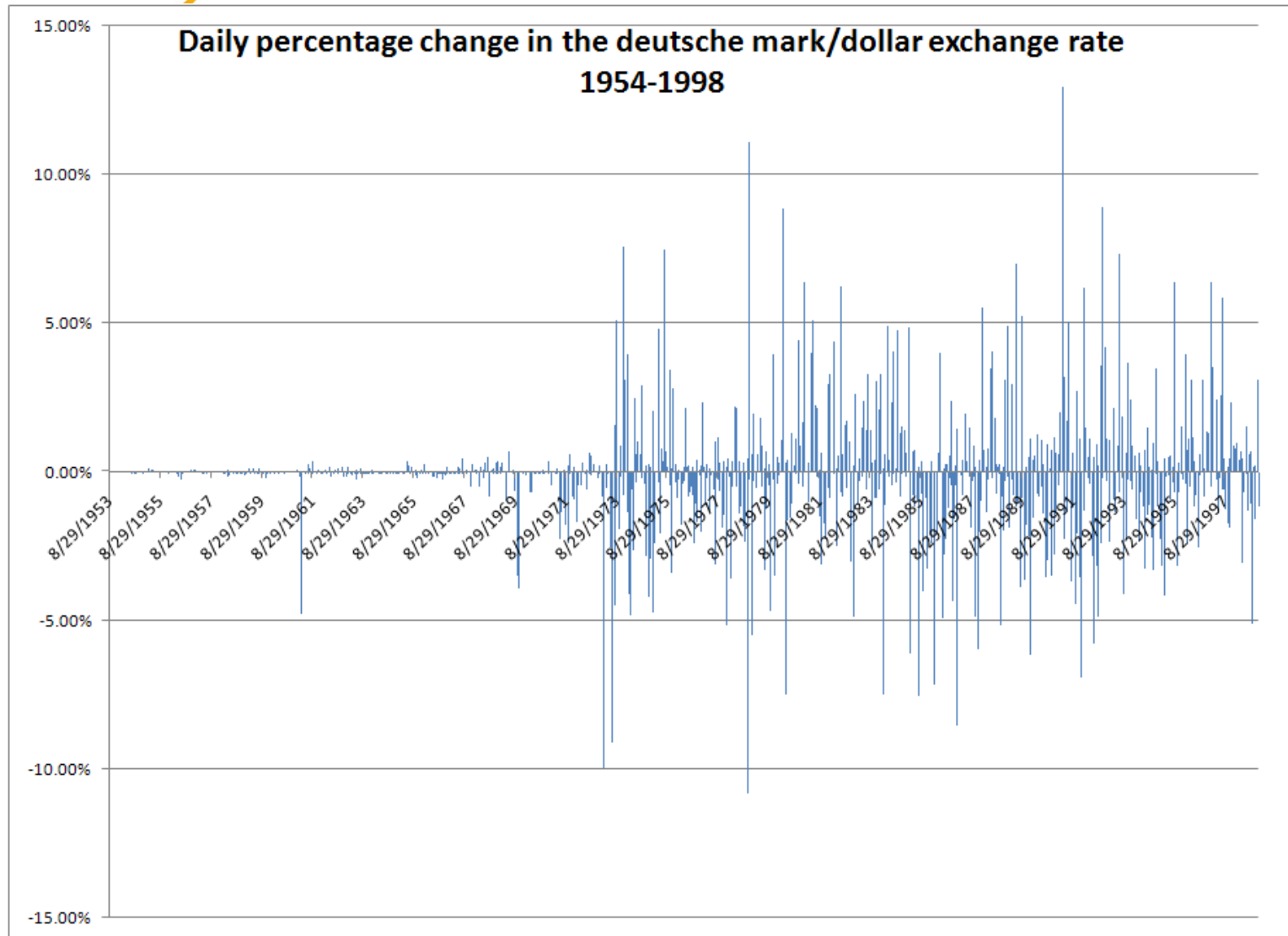
Monthly percentage change in the producer price index for oil, 1947–2004.



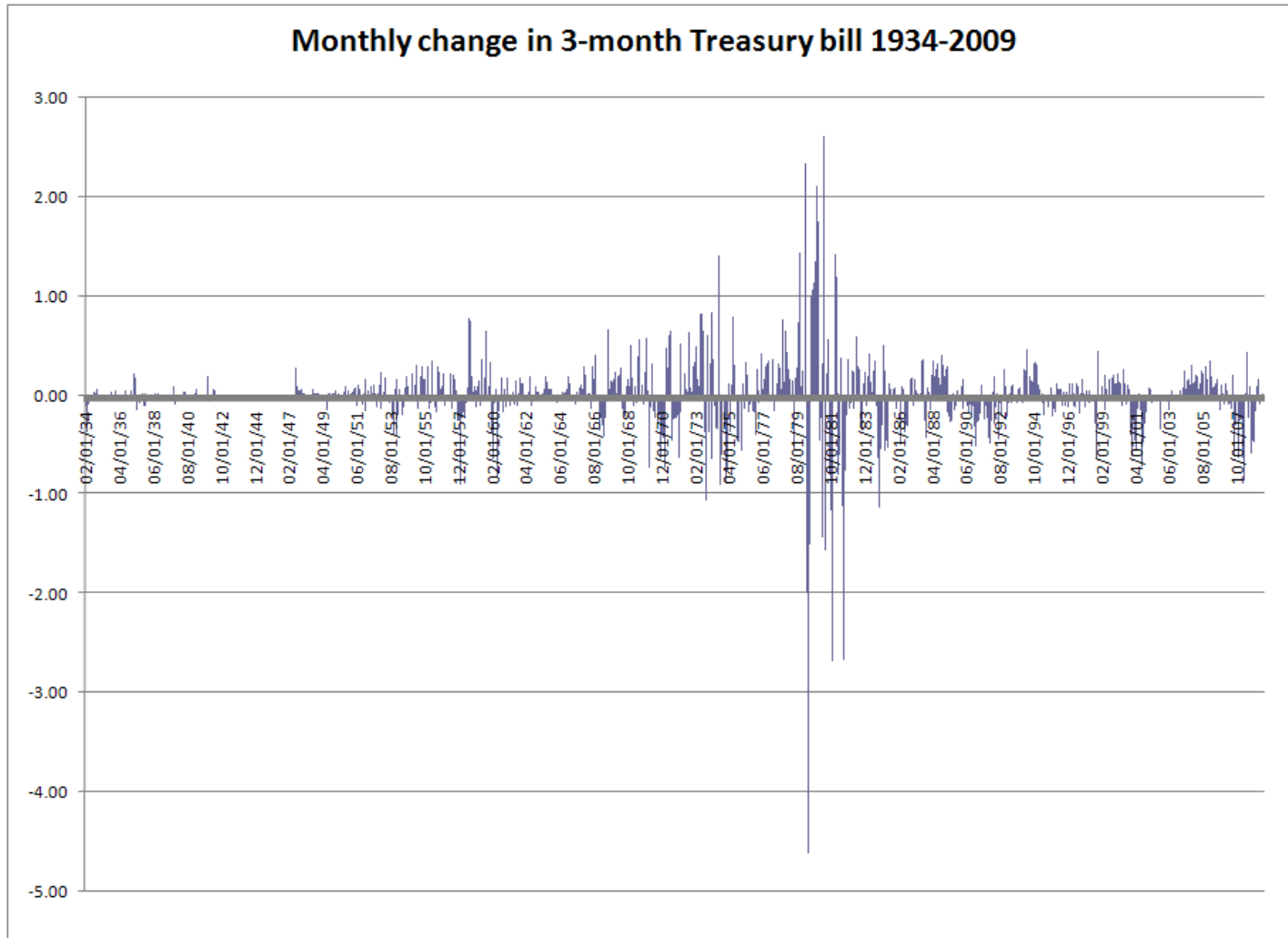
Source: DRI.

Source: McDonald *Derivatives Markets* 2d ed. Pearson 2006

Derivatives and price variability: exchange rates



Derivatives and price variability: interest rates



Forward contract: Definition

- Contract whereby parties are committed:
 - to buy (sell)
 - an underlying asset
 - at some future date (maturity)
 - at a delivery price (forward price) set in advance
- The forward price for a contract is the delivery price that would be applicable to the contract if were negotiated today (i.e., it is the delivery price that would make the contract worth exactly zero)
- The forward price may be different for contracts of different maturities

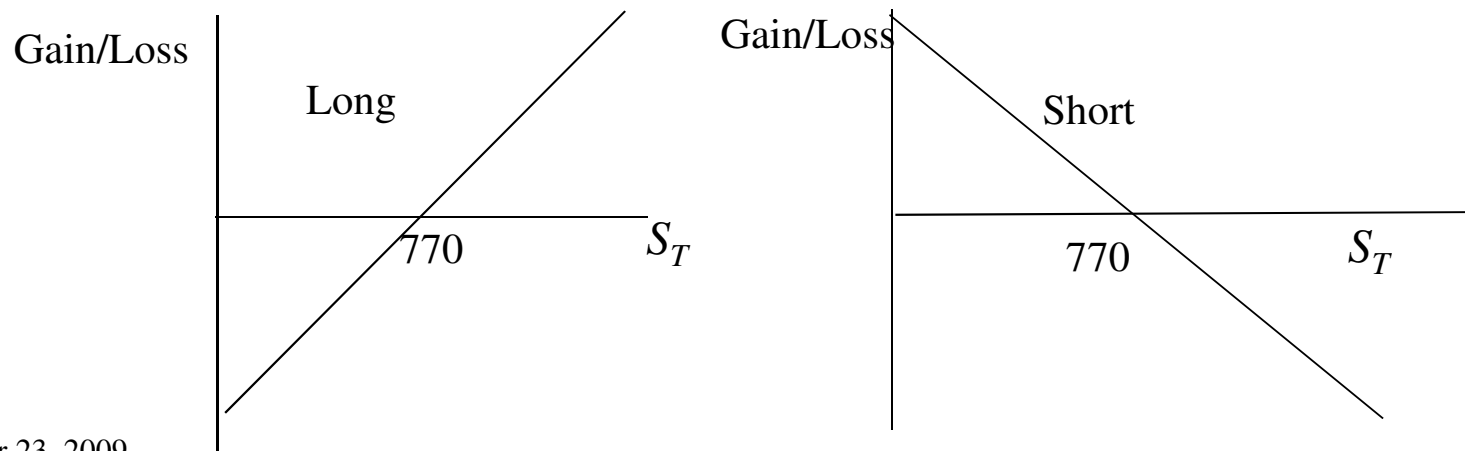
- Buying forward = "LONG" position
- Selling forward = "SHORT" position
- When contract initiated: No cash flow
- Obligation to transact

Forward contract: example

- Underlying asset: Gold
- Spot price: \$750 / troy ounce
- Maturity: 6-month
- Size of contract: 100 troy ounces (2,835 grams)
- Forward price: \$770 / troy ounce

Profit/Loss at maturity

Spot price	730	750	770	790	810
Buyer (long)	-4,000	-2,000	0	+2,000	+4,000
Seller (short)	+4,000	+2,000	0	-2,000	-4,000



Options: definitions

- A call (put) contract gives to the owner
 - the right :
 - to buy (sell)
 - an underlying asset (stocks, bonds, portfolios,..)
 - on or before some future date (maturity)
 - on : "European" option
 - before: "American" option
- at a price set in advance (the exercise price or striking price)
- Buyer pays a premium to the seller (writer)

- Exchange traded
 - Traditionally exchanges have used the open-outcry system, but increasingly they are switching to electronic trading
 - Contracts are standard there is virtually no credit risk
 - Europe
 - Eurex: <http://www.eurexchange.com/>
 - NYSE Euronext Liffe: <http://www.liffe.com>
 - United States
 - CME Group <http://www.cme.com>
- Over-the-counter (OTC)
 - A computer- and telephone-linked network of dealers at financial institutions, corporations, and fund managers
 - Contracts can be non-standard and there is some small amount of credit risk

Evolution of global market

DERIVATIVES MARKETS (futures and options)

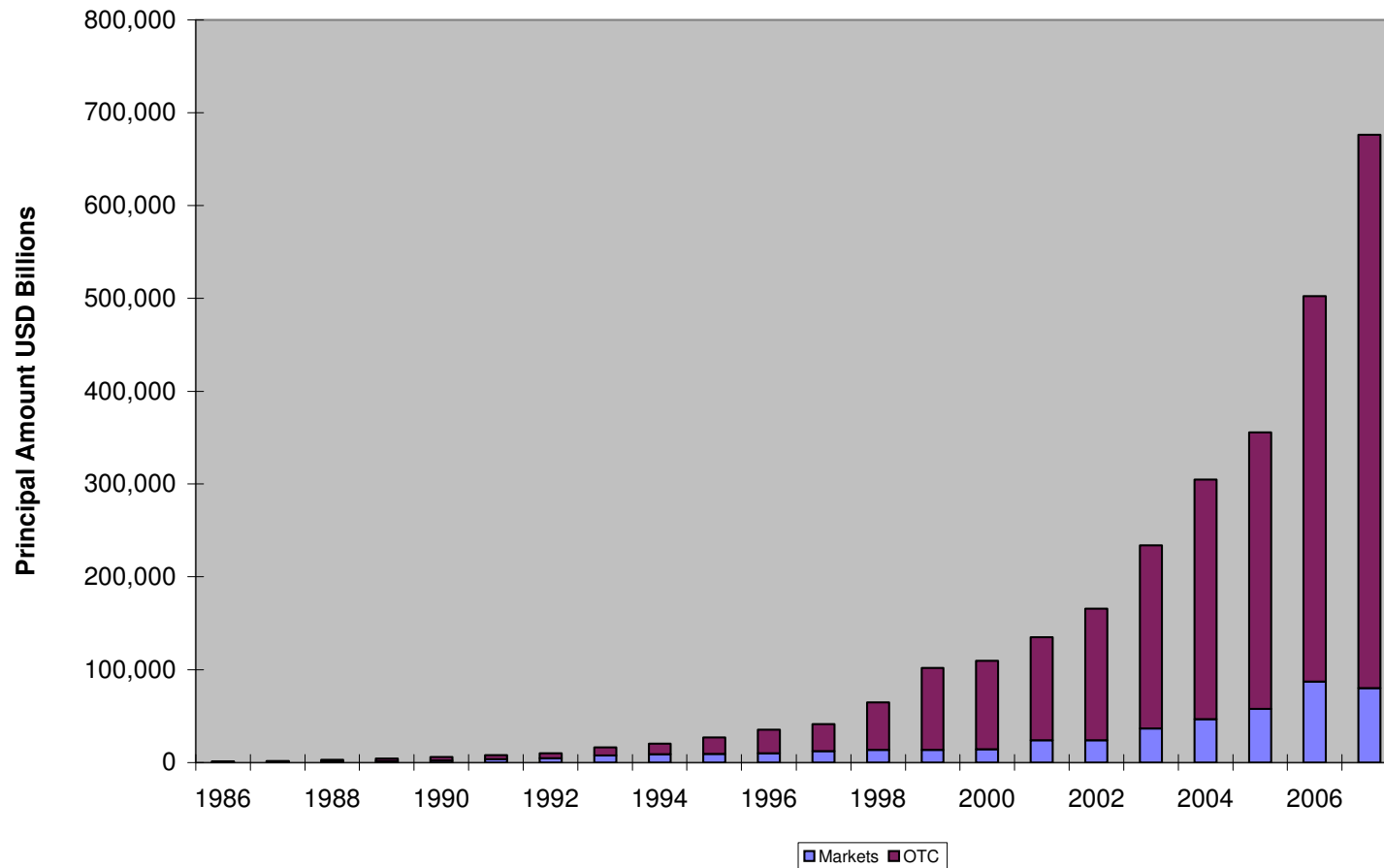


Table 19: Amounts outstanding of over-the-counter (OTC) derivatives

By risk category and instrument

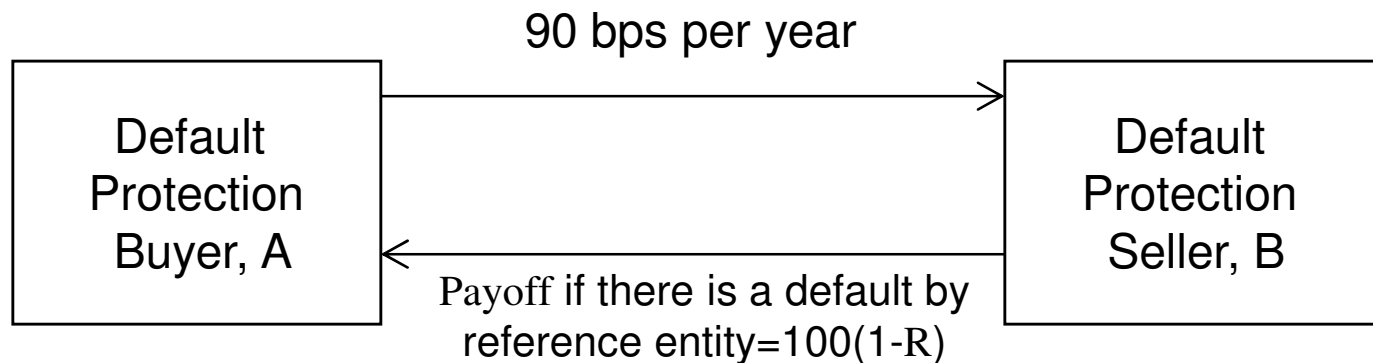
In billions of US dollars

Risk Category / Instrument	Notional amounts outstanding					Gross market values				
	Dec 2006	Jun 2007	Dec 2007	Jun 2008	Dec 2008	Dec 2006	Jun 2007	Dec 2007	Jun 2008	Dec 2008
Total contracts	418,131	516,407	595,341	683,726	591,963	9,791	11,140	15,813	20,353	33,889
Foreign exchange contracts	40,271	48,645	56,238	62,983	49,753	1,266	1,345	1,807	2,262	3,917
Forwards and forex swaps	19,882	24,530	29,144	31,966	24,562	469	492	675	802	1,732
Currency swaps	10,792	12,312	14,347	16,307	14,725	601	619	817	1,071	1,588
Options	9,597	11,804	12,748	14,710	10,466	196	235	315	388	597
Interest rate contracts	291,581	347,312	393,138	458,304	418,678	4,826	6,063	7,177	9,263	18,420
Forward rate agreements	18,668	22,809	26,599	39,370	39,262	32	43	41	88	153
Interest rate swaps	229,693	272,216	309,588	356,772	328,114	4,163	5,321	6,183	8,056	16,573
Options	43,221	52,288	56,951	62,162	51,301	631	700	953	1,120	1,694
Equity-linked contracts	7,488	8,590	8,469	10,177	6,494	853	1,116	1,142	1,146	1,113
Forwards and swaps	1,767	2,470	2,233	2,657	1,632	166	240	239	283	338
Options	5,720	6,119	6,236	7,520	4,862	686	876	903	863	775
Commodity contracts	7,115	7,567	8,455	13,229	4,427	667	636	1,899	2,209	955
Gold	640	426	595	649	395	56	47	70	68	65
Other commodities	6,475	7,141	7,861	12,580	4,032	611	589	1,829	2,142	890
Forwards and swaps	2,813	3,447	5,085	7,561	2,471					
Options	3,663	3,694	2,776	5,019	1,561					
Credit default swaps	28,650	42,581	57,894	57,325	41,868	470	721	2,002	3,172	5,652
Single-name instruments	17,879	24,239	32,246	33,334	25,730	278	406	1,143	1,889	3,695
Multi-name instruments	10,771	18,341	25,648	23,991	16,138	192	315	859	1,283	1,957
Unallocated	43,026	61,713	71,146	81,708	70,742	1,709	1,259	1,788	2,301	3,831
Memorandum Item:										
Gross Credit Exposure						2,036	2,672	3,256	3,859	5,004

Credit Default Swaps

- A huge market with over \$50 trillion of notional principal
- Buyer of the instrument acquires protection from the seller against a default by a particular company or country (the reference entity)
- Example: Buyer pays a premium of 90 bps per year for \$100 million of 5-year protection against company X
- Premium is known as the *credit default spread*. It is paid for life of contract or until default
- If there is a default, the buyer has the right to sell bonds with a face value of \$100 million issued by company X for \$100 million (Several bonds are typically deliverable)

CDS Structure (Figure 23.1, page 527)



Recovery rate, R , is the ratio of the value of the bond issued by reference entity immediately after default to the face value of the bond

Other Details

- Payments are usually made quarterly in arrears
- In the event of default there is a final accrual payment by the buyer
- Settlement can be specified as delivery of the bonds or in cash
- Suppose payments are made quarterly in the example just considered. What are the cash flows if there is a default after 3 years and 1 month and recovery rate is 40%?

BIS statistics - Organized exchanges - Futures

Table 23A: Derivative financial instruments traded on organised exchanges

By instrument and location

Notional principal in billions of US dollars

Instrument / location	Amounts outstanding				Turnover					
	Dec 2006	Dec 2007	Dec 2008	Mar 2009	2007	2008	Q2 2008	Q3 2008	Q4 2008	Q1 2009
Futures										
All markets	25,668.4	28,038.9	19,483.5	18,512.4	1,585,887.9	1,547,301.1	428,512.9	387,294.4	246,319.1	241,671.6
Interest rate	24,476.2	26,769.6	18,732.3	17,833.7	1,433,766.9	1,392,798.3	390,811.9	345,173.4	214,931.2	217,877.1
Currency	161.4	158.5	95.2	86.1	20,326.4	24,396.9	6,702.8	7,201.1	4,593.9	4,392.5
Equity index	1,030.8	1,110.8	656.0	592.5	131,794.7	130,105.9	30,998.1	34,919.9	26,794.1	19,402.0
North America	13,734.5	14,477.3	10,142.4	9,487.5	872,124.7	850,706.0	235,741.1	215,902.2	137,796.4	131,506.9
Interest rate	13,077.0	13,844.1	9,818.8	9,205.1	801,680.3	774,439.1	217,241.3	194,846.2	121,570.0	118,939.0
Currency	136.4	101.4	59.9	56.8	17,654.7	20,861.8	5,858.7	6,055.6	3,858.7	3,744.8
Equity index	521.0	531.8	263.7	225.6	52,789.8	55,405.1	12,641.2	15,000.4	12,367.7	8,823.1
Europe	8,148.9	9,012.5	6,506.5	6,622.8	588,093.0	591,030.1	163,207.5	144,159.6	91,882.9	93,984.5
Interest rate	7,801.7	8,639.5	6,252.3	6,364.6	538,134.3	543,902.3	151,927.0	131,555.5	82,943.5	87,683.2
Currency	1.8	5.7	5.3	3.2	106.6	157.7	46.0	52.6	25.7	14.0
Equity index	345.3	367.3	248.9	254.9	49,852.1	46,970.1	11,234.6	12,551.6	8,913.6	6,287.3
Asia and Pacific	3,363.3	3,943.3	2,466.5	2,022.9	107,923.9	89,259.5	24,599.2	22,851.7	14,104.6	13,259.4
Interest rate	3,210.7	3,745.5	2,327.1	1,919.4	81,497.8	63,811.5	18,223.9	16,099.6	8,829.5	9,131.7
Currency	8.1	23.7	7.9	6.9	282.2	353.6	73.1	91.0	104.3	127.6
Equity index	144.5	174.0	131.5	96.6	26,143.9	25,094.5	6,302.2	6,661.1	5,170.8	4,000.0
Other Markets	421.7	605.8	368.1	379.1	17,746.3	16,305.5	4,965.0	4,381.0	2,535.2	2,920.9
Interest rate	386.8	540.5	334.0	344.6	12,454.5	10,645.5	3,419.8	2,672.2	1,588.2	2,123.2
Currency	15.0	27.7	22.2	19.1	2,283.0	3,023.7	725.1	1,002.0	605.2	506.2
Equity index	20.0	37.7	11.9	15.4	3,008.8	2,636.3	820.1	706.8	341.9	291.6

BIS - Organized exchanges - Options

Table 23A: Derivative financial instruments traded on organised exchanges

By instrument and location

Notional principal in billions of US dollars

Instrument / location	Amounts outstanding				Turnover					
	Dec 2006	Dec 2007	Dec 2008	Mar 2009	2007	2008	Q2 2008	Q3 2008	Q4 2008	Q1 2009
Options										
All markets	43,722.0	51,039.1	38,376.4	37,245.5	702,109.2	666,043.8	171,276.5	154,748.7	133,834.7	125,316.8
Interest rate	38,116.4	44,281.7	33,978.8	33,005.8	547,629.2	526,860.8	135,367.0	113,453.6	107,742.5	106,302.9
Currency	78.6	132.7	124.8	107.3	2,140.5	2,782.0	822.4	730.6	444.9	411.3
Equity index	5,527.0	6,624.7	4,272.8	4,132.4	152,339.5	136,401.0	35,087.0	40,564.4	25,647.3	18,602.6
North America	27,770.9	28,024.2	19,672.3	17,323.5	414,892.0	323,330.4	81,601.0	74,586.7	61,141.0	47,784.5
Interest rate	24,844.4	25,084.1	17,788.9	15,523.2	363,599.0	275,452.6	69,943.3	60,652.9	51,135.6	41,462.7
Currency	32.6	57.1	40.5	48.9	621.8	898.7	243.0	255.8	161.6	151.6
Equity index	2,893.9	2,883.0	1,842.9	1,751.5	50,671.2	46,979.1	11,414.7	13,678.0	9,843.8	6,170.3
Europe	15,066.7	21,554.5	18,116.0	19,444.8	203,789.8	276,253.0	71,751.1	59,516.9	61,755.8	67,873.6
Interest rate	12,702.2	18,109.7	15,879.5	17,307.6	176,639.7	245,826.9	63,860.5	51,386.7	55,973.3	63,947.8
Currency	0.7	1.0	0.6	0.6	7.6	6.6	1.2	2.1	1.8	2.2
Equity index	2,363.8	3,443.8	2,235.9	2,136.6	27,142.5	30,419.5	7,889.4	8,128.1	5,780.8	3,923.6
Asia and Pacific	680.7	1,020.7	219.4	219.5	77,835.9	60,095.1	16,147.6	18,964.8	10,039.6	8,614.4
Interest rate	459.7	791.1	83.8	24.1	6,341.3	3,902.6	1,111.3	941.6	425.3	474.2
Currency	-	-	-	-	-	-	-	-	-	-
Equity index	221.0	229.6	135.6	195.4	71,494.6	56,192.4	15,036.2	18,023.2	9,614.3	8,140.3
Other Markets	203.8	439.7	368.7	257.8	5,591.5	6,365.3	1,776.8	1,680.3	898.3	1,044.3
Interest rate	110.2	296.7	226.6	150.9	1,049.1	1,678.7	451.9	472.5	208.3	418.3
Currency	45.4	74.7	83.7	57.8	1,511.2	1,876.7	578.2	472.7	281.5	257.5
Equity index	48.2	68.3	58.5	49.0	3,031.2	2,810.0	746.8	735.2	408.4	368.4

Why use derivatives?

- To hedge risks
- To speculate (take a view on the future direction of the market)
- To lock in an arbitrage profit
- To change the nature of a liability
- To change the nature of an investment without incurring the costs of selling one portfolio and buying another

Forward contract: Cash flows

- Notations

S_T Price of underlying asset at maturity

F_t Forward price (delivery price) set at time $t < T$

	Initiation	Maturity T
Long	0	$S_T - F_t$
Short	0	$F_t - S_T$

- Initial cash flow = 0 : delivery price equals forward price.
- Credit risk during the whole life of forward contract.

Forward contract: Locking in the result before maturity

- Enter a new forward contract in opposite direction.
- Ex: at time t_1 : long forward at forward price F_1
- At time t_2 ($<T$): short forward at new forward price F_2
- Gain/loss at maturity :
- $(S_T - F_1) + (F_2 - S_T) = F_2 - F_1$ no remaining uncertainty

Futures contract: Definition

- Institutionalized forward contract with daily settlement of gains and losses
- Forward contract
 - Buy \Leftrightarrow long
 - sell \Leftrightarrow short
- Standardized
 - Maturity, Face value of contract
- Traded on an organized exchange
 - Clearing house
- Daily settlement of gains and losses (Marked to market)

Example: Gold futures

Trading unit: 100 troy ounces (2,835 grams)



9/7/2007 Session Expanded Table

	Last	Open High	Open Low	High	Low	Most Recent Settle	Change	Open Interest	Volume
<u>Sep 2007</u>	n/a	n/a	n/a	n/a	n/a	700.80	+5.20	13	n/a
<u>Oct 2007</u>	703.50	702.50	702.00	710.00	695.10	703.20	+5.10	28996	2922
<u>Nov 2007</u>	708.00	n/a	706.50	709.00	706.50	706.40	+5.10	13	7
<u>Dec 2007</u>	710.00	709.00	708.00	716.60	701.50	709.70	+5.10	214225	106697
<u>Feb 2008</u>	716.00	n/a	709.90	720.80	708.50	715.70	+5.00	17387	3313
<u>April 2008</u>	724.40	n/a	715.20	724.50	714.60	721.30	+4.80	16825	93
<u>June 2008</u>	726.00	n/a	721.90	733.30	719.50	726.70	+4.60	15192	168
<u>Aug 2008</u>	725.90	n/a	725.90	725.90	725.90	732.00	+4.50	8707	31

Futures: Daily settlement and the clearing house

- In a forward contract:
 - Buyer and seller face each other during the life of the contract
 - Gains and losses are realized when the contract expires
 - Credit risk

BUYER ⇔ SELLER

- In a futures contract
 - Gains and losses are realized daily (Marking to market)
 - The clearinghouse guarantees contract performance : steps in to take a position opposite each party

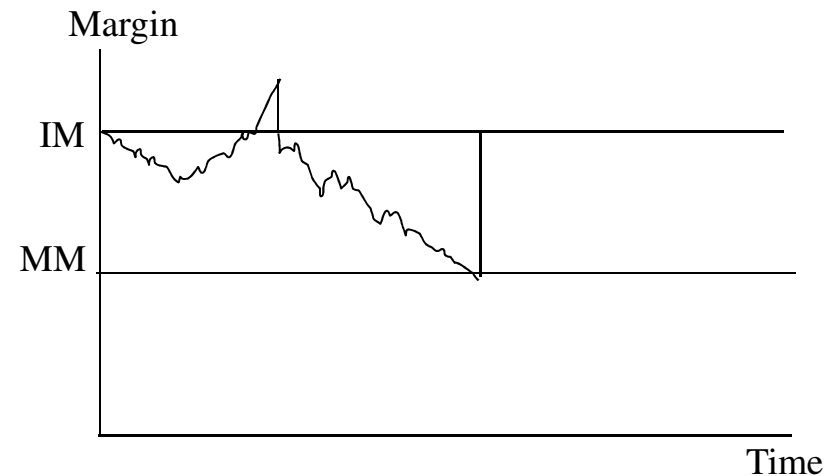
BUYER ⇔ CH ⇔ SELLER

Futures: Margin requirements

- INITIAL MARGIN : deposit to put up in a *margin account*
- MAINTENANCE MARGIN : minimum level of the margin account
- MARKING TO MARKET : balance in margin account adjusted daily

$$\text{LONG(buyer)} \quad + \text{Size} \times (F_{t+1} - F_t)$$

$$\text{SHORT(seller)} \quad - \text{Size} \times (F_{t+1} - F_t)$$



- Equivalent to writing a new futures contract every day at new futures price
- (Remember how to close of position on a forward)
- Note: timing of cash flows different

Valuing forward contracts: Key ideas

- Two different ways to own a unit of the underlying asset at maturity:
 - 1. Buy spot (SPOT PRICE: S_0) and borrow
=> Interest and inventory costs
 - 2. Buy forward (AT FORWARD PRICE F_0)
- **VALUATION PRINCIPLE: NO ARBITRAGE**
- In perfect markets, no free lunch: the 2 methods should cost the same.

You can think of a derivative as a mixture of its constituent underliers, much as a cake is a mixture of eggs, flour and milk in carefully specified proportions. The derivative's model provide a recipe for the mixture, one whose ingredients' quantity vary with time.

Emanuel Derman, Market and models, *Risk* July 2001

Discount factors and interest rates

- Review: Present value of C_t
 - $PV(C_t) = C_t \times \text{Discount factor}$
- With annual compounding:
 - Discount factor = $1 / (1+r)^t$
- With continuous compounding:
 - Discount factor = $1 / e^{rt} = e^{-rt}$

Forward contract valuation : No income on underlying asset

- Example: Gold (provides no income + no storage cost)
 - Current spot price $S_0 = \$750/\text{oz}$
 - Interest rate (with continuous compounding) $r = 5\%$
 - Time until delivery (maturity of forward contract) $T = 1$

- Forward price F_0 ? $t = 0$ $t = 1$

Strategy 1: buy forward

0

$S_T - F_0$

Strategy 2: buy spot and borrow

Buy spot

-750

$+ S_T$

Borrow

+750

-788.45

0

$S_T - 788.45$

Should
be
equal

Forward price and value of forward contract

- Forward price:

$$F_0 = S_0 e^{rT}$$

- Remember: the forward price is the delivery price which sets the value of a forward contract equal to zero.
- Value of forward contract with delivery price K

$$f = S_0 - Ke^{-rT}$$

- You can check that $f = 0$ for $K = S_0 e^{rT}$

- If $F_0 \neq S_0 e^{rT}$: arbitrage opportunity
- Cash and carry arbitrage if: $F_0 > S_0 e^{rT}$
 - Borrow S_0 , buy spot and sell forward at forward price F_0
- Reverse cash and carry arbitrage if $S_0 e^{rT} > F_0$
 - Short asset, invest and buy forward at forward price F_0

Arbitrage: examples

- Gold – $S_0 = 750$, $r = 5\%$, $T = 1$ $S_0 e^{rT} = 788.45$

- If forward price = 800
 - Buy spot -750 $+S_1$
 - Borrow +750 -788.45
 - Sell forward 0 $+800 - S_1$
 - Total 0 + 11.55

- If forward price = 760
 - Sell spot +750 $-S_1$
 - Invest -750 +788.45
 - Buy forward 0 $S_1 - 760$
 - Total 0 + 28.45